1201 (Algebra 1)

 Year:
 2014–2015

 Code:
 MATH1201

Level: First

Value: Half unit (= 7.5 ECTS credits)

Term:

Structure: 3 hour lectures and 1 hour problem class per week.

Weekly assessed coursework.

Assessment: The final weighted mark for the module is given by: 90% examination,

10% coursework. The coursework mark is obtained from exercise sheet marks (5%) and the mid-sessional examination result (5%). In order to pass the module you must have at least 40% for both the

examination mark and the final weighted mark.

Normal Pre-requisites: A* in A-level Mathematics and Further Mathematics

Lecturer: Prof FEA Johnson
Problem class teacher: Dr ML Roberts

Course Description and Objectives

1201 is intended as an introduction to the methods of modern Algebra and Discrete Mathematics, primarily through the detailed study of a problem of particular importance and wide applicability, namely linear equations. In addition, a number of other important aspects of modern mathematics are introduced, in particular, Logic, Set Theory, Functions and Mappings, Permutations, Fields.

The main aim of the course is to lay a solid foundation for the rest of the three years.

Recommended Texts

The material on linear algebra is very well covered in a large number of excellent texts. Making a specific recommendation is largely a matter of taste. However,

- (i) Guide to Linear Algebra by David Towers, published by MacMillan, covers the basics and is quite cheap.
- (ii) Elementary Linear Algebra by Howard Anton, published by Wiley, is more thorough, though still elementary.
- (iii) Linear Algebra by S. Lang, published by Wiley is more sophisticated.

The material on Logic may also be found in many places, for example:

- (i) Foundations of Mathematics by I. Stewart and D. O. Tall, published by Oxford Scientific Publications.
- (ii) Discrete and Combinatorial Mathematics by Ralph P. Grimaldi, published by Addison-Wesley.

Detailed Syllabus

Logic

Propositional calculus via truth tables. Minimal formulations $(\vee, \neg), (\wedge, \neg), (\Longrightarrow, \neg)$. Elementary Predicate Calculus.

Rules for manipulating quantifiers by reference to finite models.

Duality under \neg .

Methods of proof: Contradiction. Induction.

Linear Equations

Matrix formulation and elementary operations. Reduction to row echelon form.

Solution of system in canonical form.

Vector Spaces

Subspaces of \mathbb{R}^n , \mathbb{Q}^n : Linear independence. Basis and dimension.

Elementary Set Theory

Sets introduced via defining predicates.

Subsets, unions, intersections, complements.

Boole-De Morgan indentities.

Mappings

Injectivity. Surjectivity. Bijectivity. Invertibility.

Permutations. Decomposition into

- (i) disjoint cycles
- (ii) adjacent transpositions.

 $\operatorname{Sign}(\sigma \tau) = \operatorname{Sign}(\sigma)\operatorname{Sign}(\tau)$ via Laplace's Formula.

Linear Mappings

Linear mappings and matrices

 $\dim \operatorname{Ker} + \dim \operatorname{Im} = \dim \operatorname{Domain}$.

Fields

 \mathbb{Q} , \mathbb{R} , introduction to \mathbb{C} . Finite fields \mathbb{F}_p .

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