G508 (Financial Mathematics)

Year:	2014–2015
Code:	MATHG508
Level:	Masters
Value:	15 UCL credits (= 6 ECTS credits)
Term:	1
Structure:	2 hour lectures and 1 hour problem class per week. Compulsory assessed coursework.
Assessment:	90% examination, 10% coursework
Lecturer:	Dr J Walton
Problem class teacher:	TBA

Course Description and Objectives

This course is an introduction to applying mathematics to the study of instruments used in finance. It is primarily concerned with the valuation of financial derivatives.

What will the course achieve? A review of a variety of financial contracts involving payments at a future date. Definition and examples of hedging strategies (a form of insurance against risk). Explanation of: (i) pricing based on the assumption of no arbitrage opportunities, and (ii) of the Stochastic model of share price changes. (This requires some elementary probability which we revise.) Introduction of tools from calculus and from linear algebra in order to: (i) deduce the Black-Scholes equation and the risk-neutral valuation principle, (ii) solve some simple partial differential equations arising in finance, (iii) enable the determination of the synthetic probability distribution of share prices at a future date, (iv) establishment of discrete models in particular the Binomial model. Calculation of the value of European put and call options (the Black-Scholes formula) using continuous time calculus and using discrete time approximation. Hedging approaches for options including their "greeks".

Recommended Texts

The main texts referred to are: M Baxter, A Rennie, *Financial Calculus*, CUP, 1996, ISBN 0521-55289-3; J C Hull, *Options, Futures and other Derivatives*, Prentice Hall, 1989, ISBN 013-264367-7.

Background texts are: S R Pliska, *Introduction to Mathematical Finance-Discrete Time Models*, Blackwell, 1997, ISBN 1-55786945-6; R C Merton, *Continuous-time Finance*, Blackwell, 1996, ISBN 0-631-18508-9; P L Bernstein, *Capital Ideas – The Improbable Origins of Modern Wall Street*, Free Press, 1992.

Detailed Syllabus

Introduction to forwards, futures and options. Foreign exchange and covered interest rate parity. Representation of asset prices by vectors. Basic ideas from probability (expected value, distributions, densities, conditional expectation, covariance as inner-product). Pricing claims in a complete market. No-arbitrage theorem. European options. American options. One period tree model. Mathematics of the single–period model. Linear pricing measure. Law of One Price. Arbitrage Opportunities. Risk-neutral measures. Proof of the No-arbitrage Theorem. Calculating the value of a contingent claim. Multi-period models. Partitions and filtrations. Conditional probability. Measurable processes. Adapted processes. Predictable self-financing strategies. Law of iterated expectation. Multi-period no-arbitrage theorem: construction of a

measure from the one-period theorem. Gains process is a martingale. Converse: non-existence of arbitrage opportunities in the presence of a risk-neutral measure. Pricing American options: Stopping times, Dynamic hedging strategy. Binomial model in general multi-period calculations. Continuous Time Model. Justifying $\text{var}(dz) = dt$. Normality assumption versus binomial assumption. Ito's Lemma. Black-Scholes Equation. Kac-Feynman Formula. Black-Scholes Formula for a call option. Delta and the greeks for European options.