MATHGM02 (Nonlinear Systems)

Year:	2016–2017
Code:	MATHGM02
Level:	Masters
Value:	15 UCL credits (= 6 ECTS credits)
Term:	1
Structure:	3 hours lectures per week
Assessment:	100% examination. Weekly Homework Problem Sheets for practice (marked and returned to provide feedback).
Lecturer:	Professor SR Bishop

Course Description and Objectives

This component aims to give an overview of the main aspects of nonlinear systems. It will provide basic definitions and establish theoretical background. Where possible, practical applications will be considered via a hands-on approach to build up ability to model and visualise dynamical models.

Recommended Texts

(i) S.H.Strogatz, *Nonlinear Dynamics and Chaos*, Perseus Books, 1994.

(ii) J.M.T.Thompson & H.B. Stewart, *Nonlinear Dynamics and Chaos*, Wiley 2003.

(iii) E.Ott, *Chaos in Dynamical Systems*, CUP, 1993.

(iv) D.K.Arrowsmith & C.M.Place, *Dynamical Systems*, Chapman Hall 1990.

(v) J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields*, Springer 1983.

(vi) P.Drazin *Nonlinear Systems*, Cambridge University Press, 1992.

(vii) P.Drazin and R.S.Johnson, *Solitons*, Cambridge University Press, 1989.

(viii) L.D.Landau & E.M.Lifshitz, *A Course of Theoretical Physics*, Vol. 1 Mechanics, Pergamon 1960.

(ix) E.Infeld & G.Rowlands, *Nonlinear Waves, Solitons and Chaos*, CUP, 2000.

Detailed Syllabus

Introduction: Review of dynamics and modelling concepts. The role of nonlinearity and damping. Deterministic versus probabilistic models.

Continuous Dynamical Systems: Flows Governed By ODEs Transients and steady states - equilibria, periodic and chaotic solutions. Local and global stability. Liouvilles Theorem. Conservative and dissipative mechanical systems. Periodic solutions and Poincare-Bendixson theorem. Link to perturbation methods. Bifurcation theory for 1- and 2-dimensional systems including structural stability of bifurcations. Comparison of Hamiltonian systems and dissipative systems. Chaos and the butterfly effect. Potential well dynamics for nonlinear oscillators. Numerical considerations including basins of attraction, the role of unstable saddles, homoclinic trajectories and Lyapunov exponents. Link to flows via Poincare maps.

Discrete Dynamical systems: Maps Iterated maps as dynamical systems in discrete time. The logistic map as main example. Equilibria, cycles and their stability. Period doubling and other bifurcations of maps. Elementary properties of maps in two dimensions. Lyapunov exponents for maps.

Linear waves, dispersion relations, dissipation leading to stable and unstable waves. Travelling wave solutions of non-linear partial differential equations, for example the Korteweg-de Vries, non-linear Schrodinger equations. Solitons and links with homoclinic orbits of dynamical systems.