MATHGF01 – Numerical Analysis for Finance

Tutor:
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Aims:
This course will introduce the students to Numerical Analysis with emphasis on the techniques used in the pricing and risk management systems used by a derivatives trading desk.

Objectives/outcomes:
Demonstrable skills in applying numerical schemes and programming proficiency in basic C++ to solve practical problems in Mathematical Finance.

Content:

- **Introduction:** Sampler on some typical numerical problems encountered in quantityative finance: vol from premium, strike from delta, derivatives pricing equations, calibration of SVOL models, pathologies
- **Numerical root finding:** bracketing, bisection, secant, regula falsi, Newton Raphson, Householder, Ridders, Brent, error estimates, strike from delta, vol from premium, internal rate of return, DNTs with given TV
- **Interpolation/Extrapolation:** Linear, polynomial fits, Runge's phenomenon, ill-conditioning of the Vandermonde matrix, Pade approximants, splines, local splines, monotonic interpolation. Interpolations of vols and rates.
- **Some numerical issues arising in Stochastic Volatility models**: golden ratio, steepest descent, Calibration of Stochastic Volatilty models, quadratures.
- **Pricing on Trees:** binomial, trinomial trees, convergence, Richardson extrapolation, Greek calculation, pricing of American options
- **Finite Difference Grids for the resolution of pricing PDEs:** implicit, explicit, Crank Nicholson, Fourier modes, stability and convergence, Lax equivalence, ADI, Craig Snyed, pricing of barriers in presence of vol and rates term structure
- **Numerical linear algebra:** LU decomposition, Jacobi, Gauss Seidel, SOR, sparse matrices.
- **Monte Carlo methods:** Generation of random numbers, Law of large numbers, central limit theorem, random number generation, from uniform to other distributions, Box Muller, strong/weak convergence, Euler-Maruyama, Milstein, simulation of GBM, Excel demonstration of the Ito formula, pricing of some derivatives, antithetics, control variates, Multidimensional Monte Carlo, the curse of dimensionality, Brownian bridge, Choleski, correlation matrices, low discrepancy numbers, simulation of stochastic volatility models.
- **Copulae:** definition, Sklar theorem, examples and typical uses, caveats, pricing of bivariate digitals, robust correlation statistics.
- **Time series analysis:** estimation of vol and correlation, stability issues, lags, autocorrelation, confidence intervals, robust statistics

Recommended text:

1. Iain J. Clark, Foreign exchange option pricing: A practitioner's guide, Wiley Finance 2011.
2. Germund Dahlquist and Ake Bjork, Numerical Methods Prentice-Hall series in automatic computation, 1974, reprinted by Dover 2004.
3. Paul Glasserman, Monte Carlo Methods in Financial Engineering, Springer Verlag 2003
4. Peter Jaeckel, Monte Carlo methods in Finance Wiley Finance 2002
5. K. W. Morton and D. F. Mayers, Numerical Solution of partial differential equations, Cambridge University Press 2011

6. William H. Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery, Numerical Recipes in C++. The Art of Scientific Computing Cambridge University Press 2002

Structure:

20 hours of lecture presentation plus 10 hours of practical work.

Assessment:

Written Examination (2 hours)

To pass this course, students must: Obtain an overall pass mark of 50%.