**Introduction to advanced learning algorithms in artificial neural networks**

**AIMS**

- Discuss the use of multi-layered networks to overcome the limitations of the perceptron, and possible applications of such networks.
- Explain the problems posed to learning by the credit assignment problems caused by correct responses not being provided for each neuron, or for each time-step or input stimulus.
- Discuss how reinforcement learning and genetic algorithms overcome the problems of temporal credit assignment.
- Discuss the relative biological plausibility of these learning algorithms.

**READING**

Books 1, 2, 4, 6, 14.

**Non-linearly separable problems**

Perceptrons perform linear discrimination and can’t solve ‘non-linearly separable’ problems (Minsky & Papert, 1969)

\[ w_i \rightarrow w_i + \varepsilon x_i \Delta o \]

‘Hidden’ processing units can give more power, e.g:

\[
\begin{align*}
T &= 0.5 \\
T &= 1.5 \\
T &= 0.5 \\
T &= 1.5
\end{align*}
\]

\[
\begin{array}{c|cc}
& 0 & 1 \\
0 & 0 & 1 \\
+1 & 1 & 0
\end{array}
\]

e.g. XOR ‘exclusive OR’
Multi-layer perceptrons

Q: How train connection weights if the ‘internal representation’ is not known (i.e. $t^n$ not known for ‘hidden layer’, $x^n$ not known for output?)

A: if the neurons use a smooth (continuous) transfer function, changing a connection weight from any (active) neuron anywhere in the network will change the output firing rate slightly. If the output $o^n$ is now closer to its target value $t^n$ then the change was good..

But note: The effect of changing a connection weight depends on the values of the errors $\delta^n=(t^n - o^n)$, connections $w_{ij}$ and activations $v_i$ further up in the network..

Delta rule:

$$w_{ij} \rightarrow w_{ij} + \varepsilon x^j (t^n - o^n)$$

Example:

If $w$ decreases the error for input pattern $n$ reduces

Rumelhart, Hinton & Williams (1986).

The ‘generalised delta rule’ or ‘error back-propagation’

- Present input pattern $n$ work out all other activations (a ‘forward’ pass)
- Work out the errors made by the outputs;
  $$\delta^n = (t^n - o^n)$$
- Work out the contributions of units in layer lower down to output errors (a ‘backward’ pass).
  This depends on connections $w_{ij}$ to output, giving a ‘delta’ for that layer: $\delta^2 = \sum_i w_{ij} \delta^1$.
  Similarly, for the next layer: $\delta^4 = \sum_j w_{jk} \delta^3$.
- Change connection weights in each layer using the delta rule.
- Repeat for next pattern.
- Repeat for whole training set many times.

This rule is equivalent to working out the total squared error on the whole set of training patterns: $E = \sum_n \sum_i (t^n - o^n)^2$ and changing each weight to reduce this.

Rumelhart, Hinton & Williams (1986).
Problems with the ‘generalised delta rule’ or ‘error back-propagation’

- **Local minima in error**: you can’t be sure it will find the best set of weights – maybe just finds those that can’t be improved by any small change.
- Learning and generalisation depends on the choice of architecture
- Not biologically plausible since the connection weights are changed using non-local information.

However, with enough neurons and enough training data (and recent increases in processing power), these networks outperform all other methods in some tasks.

Image classification with deep convolutional neural networks

Krizhevsky et al (2012) trained large deep network (9 layers, 750K neurons, 60M wts) to recognise 1000 categories in large training set of 1.2 M labeled images (ImageNet, 224x224xRGB). Error B-P, 90 cycles through training set x 2048 translations and reflections (6 days on 2 GPUs).

The network significantly outperformed all alternatives in categorising new images.

Image classification with deep convolutional neural networks

Example outputs


Image classification with deep convolutional neural networks

Test images

Training images with most similar activity in last hidden layer

Image classification with deep convolutional neural networks

Example of convolutional connection weights learned in first hidden layer

..bears some similarity to neural responses in the early stages of the visual system.


Deep convolutional neural networks – details that make them work

‘Convolution’: use BP, but make all patches of weights identical (average weight changes across corresponding connections in each patch), each sheet of neurons codes for the same “feature” independent of location in the image.
‘Pooling’: take the max or average output of local units (smoothing/shrinking representation)
‘Dropout’: set randomly set neurons = 0 in 50% of trials during training, use all x0.5 during testing – learns more robust sets of connections, reduces ‘overfitting’
Threshold linear transfer function: f(h) = h if h>0, f(h) = 0 if h<0. Speeds up learning.
‘Augmented dataset’: 2048 translations and reflections; random noise in pixel values on each cycle - improves ‘generalisation’.

Enough layers, enough training data, enough processing power.
Example of a sensorimotor control model: using error back-propagation.

Problem: generate action signal to achieve a desired state from given current state (e.g. play a stroke in tennis).

1. Train upper network to predict the (sensory) effect of an action signal (i.e. a forward model). Use B-P of the difference between predicted and observed effect.

2. Train the lower network to produce the correct action signal to achieve a desired (sensory) effect (an inverse model). Fix the connection weights in the upper network to use it as feedback controller. Use B-P of the difference between desired and observed effect to change weights in the lower network.


Introduction to reinforcement learning

Critic provides single reward/penalty signal $r^n$ for each output $o^n$ - but not for each output unit: a ‘credit assignment problem’.

Associative reward-penalty ‘ARP’

Output units are probabilistic: $o_i=0$ or 1, where $f(h_i)$ is the probability of $o_i = 1$. Allows exploration of possible output values.

$A_{RP}$

A ‘critic’ provides feedback:

$r^n = \begin{cases} 
1 \text{ ‘reward’} & \text{if } r^n=1 \\
-1 \text{ ‘penalty’} & \text{otherwise}
\end{cases}$ according to $o^n$

Define ‘target’ output:

$t^n = \begin{cases} 
o_i^n & \text{if } r^n=1 \\
1-o_i^n & \text{if } r^n=-1
\end{cases}$

and use ‘delta rule’: $w_{ij} \rightarrow w_{ij} + \varepsilon x_j \delta_j^n$ where $\delta_i^n = (t_i^n - f(h_i^n))$

i.e. if doing well: use delta rule to ‘reinforce’ current $h_i$ to make current output more likely, otherwise ‘penalise’. 
Using $A_{RP}$ to navigate

4 outputs: actions N, S, E, W
Critic: ‘did I get closer to tree?’ (via smell).

+ve connections  -ve connections
Initially random connections

Fig. 2. A spatial environment consisting of a central landmark (shown as a tree) surrounded by four other landmarks (shown as disks). Each landmark possesses a distinctive “odor” which can be sensed at a distance. Odor distributions decrease linearly from their associated landmarks and become undetectable at ellipses. The asterisk shows the location of the ASN.

Fig. 3. The ASN controlling locomotion in the spatial environment. The five input pathways are labelled vertically on the left according to the landmarks to which they respond. The shaded input pathway N indicates that the ASN is near the north neutral landmark. The four output pathways controlling actions are labelled horizontally at the bottom according to the direction of movement they cause. The shaded output elements indicate that a southeast movement is being made. The associative matrix weights are displayed as circles centered on the intersections of the horizontal input pathways and vertical output pathways. Positive weights are shown as hollow circles, and negative weights are shown as solid circles.

Using $A_{RP}$ to navigate, cont.

Before exploration:

The network learns the input-output associations required to move towards the goal from all start positions.

After exploration:
Introduction to reinforcement learning, cont.

• The critic in the example of $A_{RP}$ has an easy task due to the smell of the goal (tree): it is easy to know if a move has been good or bad (smell gets stronger or weaker).

• The general problem of reinforcement learning is to develop a ‘critic’ network that learns to provide feedback at each time-step even though the environment provides feedback rarely (e.g. when the tree is reached), i.e. how do you know which outputs were good or bad: the problem of ‘temporal credit assignment’.

• This involves developing an idea of the total expected future reward, and how this changes after making an action (i.e. if it changes for the better or for the worse).

• A role for the Dopamine system? – see Lecture on Reinforcement Learning.

Intro. to ‘temporal difference’ or ‘sequential reinforcement’ solution to the temporal credit assignment problem

In $A_{RP}$ the strength of the smell of the tree provides reinforcement $r(t)$ to evaluate for any action (i.e. output $o(t)$): does it leads closer to the goal or not?

If reinforcement $r(t)$ is intermittent (e.g. only when goal is reached), a ‘critic’ learns an ‘evaluation function’: the value $v$ of each state (or input) $u$ is the expected future reward from that state (given how actions are usually made).

The change in value $v(t+1) - v(t) + \delta(t)$ is used to evaluate any action $o(t)$ so output weights can be modified as in $A_{RP}$ (using $\delta(t) = v(t+1) - v(t) + r(t)$: if $\delta(t) > 0$, action $o(t)$ was good).

But how to learn $v$? A simple learning rule creates a set of connection weights $w$ so that $v(t) = w u(t)$.

This is: $w \rightarrow w + \varepsilon \delta(t) u(t)$, i.e. you can also use $\delta$ to learn weights for the critic!
Introduction to temporal difference learning (spatial example).
Use $\delta(t) = v(t+1) - v(t) + r(t)$ to evaluate actions
Use $v(t) = w \cdot u(t)$ where $w \rightarrow w + \varepsilon\delta(t)u(t)$ to learn value function.
Dopamine signals $\delta(t)$?

Temporal difference learning is a slow ‘trial and error’ based method. Development of own value function overcomes problem of temporal credit assignment.

Initial actions will be random (takes a long time to get to goal), actions improve as a result of learning in the Arp action network (but a random element is required to enable ongoing improvement: i.e. finding new shorter routes).

Introduction to evolutionary algorithms

Simplest model:
• ‘Genetic’ code determines connection weights
• Simulate behaviour of several networks with random weights over long period,
• ‘Reproduce’ most successful networks with small random changes to weights
• Repeat for many ‘generations’.

More complex models:
• Include numbers & locations neurons, rules for development in genetic code.
• Sexual reproduction of codes.
• Allow for non-transferable weight modification (learning) during life..

Paternello & Carnevali’s ‘ameoba’ (1989)

Threshold logic units (0/1 output)
Connections found by genetic algorithm.
Sensory input1: ‘see’ food ahead
Sensory input2: ‘smell’ food near
Motor output1: left-leg, (0,0)=rest, (1,0)=L
Motor output2: right-leg, (0,1)=R, (1,1)=forward

Learns to find food in a grid world after 100s of generations.
No. of offspring depends on amount of food eaten during life.
Pattern of connectivity varied.
After many generations, it learns to search in loops of the same size as the typical patches of food.

It develops various types of processing units including a ‘clock’.