Adapting random-instance sampling variance estimates and Binomial models for random-text sampling
Sean Wallis, Survey of English Usage, University College London

1. Introduction

Conventional stochastic methods based on the Binomial distribution rely on a standard model of random sampling whereby freely-varying instances of a phenomenon under study can be said to be drawn randomly and independently from an infinite population of instances.

These methods include confidence intervals and contingency tests (including multinomial tests), whether computed by Fisher’s exact method or variants of log-likelihood, $\chi^2$, or the Wilson score interval (Wallis 2013). These methods are also at the core of others. The Normal approximation to the Binomial allows us to compute a notion of the variance of the distribution, and is to be found in line-fitting and other generalisations.

In many empirical disciplines, samples are rarely drawn “randomly” from the population in a literal sense. Medical research tends to sample available volunteers rather than names compulsorily called up from electoral or medical records. However, provided that researchers are aware that their random sample is limited by the sampling method, and draw conclusions accordingly, such limitations are generally considered acceptable. Obtaining consent is occasionally a problematic experimental bias; actually recruiting relevant individuals is a more common problem.

However, in a number of disciplines, including corpus linguistics, samples are not drawn randomly from a population of independent instances, but instead consist of randomly-obtained contiguous subsamples. In corpus linguistics, these subsamples are drawn from coherent passages or transcribed recordings, generically termed ‘texts’. In this sampling regime, whereas any pair of instances in independent subsamples satisfy the independent-sampling requirement, pairs of instances in the same subsample are likely to be co-dependent to some degree.

To take a corpus linguistics example, a pair of grammatical clauses in the same text passage are more likely to share characteristics than a pair of clauses in two entirely independent passages. Similarly, epidemiological research often involves ‘cluster-based sampling’, whereby each subsample cluster is drawn from a particular location, family nexus, etc. Again, it is more likely that neighbours or family members share a characteristic under study than random individuals.

If the random-sampling assumption is undermined, a number of questions arise.

- Are statistical methods employing this random-sample assumption simply invalid on data of this type, or do they gracefully degrade?
- Do we have to employ very different tests, as some researchers have suggested, or can existing tests be modified in some way?
- Can we measure the degree to which instances drawn from the same subsample are interdependent? This would help us determine both the scale of the problem and arrive at a potential solution to take this interdependence into account.
- Would revised methods only affect the degree of certainty of an observed score (variance, confidence intervals, etc.), or might they also affect the best estimate of the observation itself (proportions or probability scores)?

2. Previous research

2.1 Employing rank tests

One approach, suggested by Vaclav Brezina and others (see e.g. Brezina and Meyerhoff 2014), is to dispense with Binomial models entirely, on the basis that they employ an assumption that is unsafe. Brezina suggests using statistics premised on rank order correlations. In examining the frequency of
particular lexical items (words) per text, he successfully demonstrates that standard Binomial models are unreliable for this reason, and proposes the Mann-Whitney test in place of $\chi^2$.

This approach has three fundamental drawbacks.

- It dispenses with a great deal of information from the sample (actual scores are replaced by rankings and instances by whole texts), so the method is necessarily conservative. (It is axiomatic in statistics that where one sound model is premised on less information than another, the lower-information model will need to err on the side of caution and be more conservative.)

- By dispensing with the assumption that data is Binomial, the method is of limited generality. The Mann-Whitney test compares two grouped subsamples, but not multiple groups on multiple dimensions (cf. $\chi^2$). Although it might conceivably be feasible to extend the test in this way, this is not a commonplace.

- Results are also of limited utility. We lose the ability to cite, with confidence, the true rate of an observed proportion or probability. This is because the test does not compare two observed rates of occurrence, but tests instead if the scores in one group of texts tend to have a significantly higher rank than those in another group. We may be able to infer that this observed ranking difference is probably the result of a higher true rate in the population from which samples are drawn. But we cannot cite any estimate of this true rate.

2.2 Case interaction models

At the other end of the information spectrum lies a method that attempts to mine as much information as possible from the sample. A corpus linguistics approach we have experimented with, and which remains a work in progress at the time of writing, attempts to estimate the interdependence between instances within subsamples by closely examining their source texts.

This is a theory-led approach. In the case of grammatical clauses, which we use as an exemplar below, linguistically we can anticipate that clauses are to be found

- embedded within other clauses to varying degrees of depth,
- found in sequences at varying distances, and
- in a co-ordinated list (for non-linguists: in a sequence with ‘and’, ‘or’, ‘but’, etc.).

In a parsed corpus, such as ICE-GB (Nelson, Wallis and Aarts 2002), not only can we obtain reliable counts of clauses but we can also obtain measures of grammatical proximity along these varying axes.

This ‘case interaction’ approach (Wallis and Aarts 2007) aggregates data from all instances to construct a model, and then estimates a prior probability score for every instance. Methods based on counting frequencies are adjusted to simply sum prior probabilities.

Compared to the method of this paper, the case interaction approach has an important potential benefit. In building this model, we first gather detailed information regarding the degree to which specific instances within a text (subsample) interact with others. This is potentially a valuable research method in its own right. In linguistics, studying the interaction between lexical and grammatical decisions (i.e. where the likelihood of a subsequent choice by a speaker being of a particular type is affected by an earlier choice) offers a route into psycholinguistic models of priming and spreading activation.

Note that not all sources of interdependence in a passage will be psycholinguistic. A range of contextual influences on co-occurring clauses, from topics of discussion to semantic and social biases, will tend to affect them simultaneously.
A final potential advantage of the case interaction approach is that with sufficient data it should be possible to control for these alternative sources of interaction (i.e. focus on psycholinguistic causes alone). From the purely stochastic point of view of this paper, however, such an ideal psycholinguistic model, by excluding these other sources of interaction, would still tend to overestimate the independence of instances and the significance of results. Therefore we see this research programme as complementary for what follows.

3. Adjusting the Binomial model

We will employ a method related to ANOVA and F-tests, applying this method to a probabilistic rather than linear scale. Consider an observation \( p \) drawn from a number of texts, \( t \), based on \( n \) total instances. Conventionally we would assume that these \( n \) instances are randomly drawn from an infinite population, and then employ the Normal approximation to the Binomial distribution:

\[
\text{standard deviation } s \equiv \sqrt{p(1 - p)/n},
\]

\[
\text{variance } s^2 \equiv p(1 - p)/n, \text{ and}
\]

\[
\text{Wilson interval } (w^-, w^+) \equiv \left( p + \frac{z_{\alpha/2}^2}{2n} \pm z_{\alpha/2} \sqrt{\frac{p(1 - p)}{n} + \frac{z_{\alpha/2}^2}{4n^2}} \right)/\left(1 + \frac{z_{\alpha/2}^2}{n}\right),
\]

where \( z_{\alpha/2} \) is the critical value of the Normal distribution for a given error level \( \alpha \) (see Wallis 2013 for a detailed discussion). Other derivations from (1) include \( \chi^2 \) and log-likelihood tests, least-square line-fitting, and so on. The model assumes that all \( n \) instances are randomly drawn from an infinite (or very large) population. However, we suspect that our subsamples are not equivalent to random samples, and that this sampling method will affect the result.

To investigate this question, our approach involves two stages. First, we measure the variance of scores between text subsamples according to two different models, one that presumes that each subsample is a random sample, and one calculated from the actual distribution of scores. The simplest method uses the frequency distribution of probability scores, \( p_i \), across all \( t \) texts, which is centred on the mean probability score of the subsamples, \( \bar{p} \). An example distribution of this type is illustrated by Figure 1.

\[
\text{subsample mean } \bar{p} = \frac{\sum p_i}{t}.
\]

If subsamples were randomly drawn from the population, it would follow from (1) that the variance could be predicted by

\[
\text{between-subsample variance } S_{ss}^2 = \frac{\bar{p}(1 - \bar{p})}{t}.
\]

To measure the actual variance of the distribution we employ a method derived from Sheskin (1997: 7). The variance of a series of \( N \) observed scores \( X_i \), may be obtained by

\[
\text{0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16 0.18 0.2}
\]

\[
\text{0 2 4 6 8 10 12}
\]

\[
\text{0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16 0.18 0.2}
\]

\[
\text{f}
\]

\[
\text{direct conversations}
\]

\[
\text{p(inter)}
\]

\[
\text{Figure 1: A frequency distribution of mean } p \text{ per text, quantised to two decimal places, tends to the Binomial distribution. See Section 4.}
\]

Adapting variance for random-text sampling

Sean Wallis
In our case, we have $t$ observed probability scores $p_i$, thus we can rewrite this formula as

$$s_{ss}^2 = \frac{\sum (p_i - \bar{p})^2}{(t-1)}.$$  

Second, we adjust the weight of evidence according to the degree to which these two variances (equations (3) and (4)) disagree. If the observed and predicted variance estimates coincide, then the total set of subsamples is, to all intents and purposes, a random sample from the population and no adjustment is needed to sample variances, standard deviations, confidence intervals, tests, etc.

We can expect, however, that in most cases the actual distribution has greater spread than that predicted by the randomness assumption. In such cases, we employ the ratio of variances, $F_{ss}$, as a scale factor for the number of random independent cases, $n$.

Gaussian variances with the same probability $p$ are inversely proportion to the number of cases supporting them, $n$, i.e. $s^2 \equiv p(1-p)/n$ (cf. equation (1)). Assuming the Normal approximation to the Binomial holds for the distribution of $\bar{p}$, we can estimate a corrected total independent sample size $n'$, by multiplying $n$ by the ratio of the variances for $\bar{p}$.

$$cluster-adjustment\ ratio\ F_{ss} = \frac{s_{\bar{p}}^2}{s_{ss}^2}, \text{ and}$$

$$corrected\ sample\ size\ n' = n \times F_{ss}.$$  

To put it another way, the ratio $n':n$ is the same as $s_{ss}^2:s_{ss}^2$. This ratio should be less than 1, and thus $n$ is decreased. If we decrease $n$ in equations (1) and (2), we obtain larger estimates of sample variance and wider confidence intervals. An adjusted $n$ is easily generalised to contingency tests and other methods.\(^1\)

In order to evaluate this method, we turn to some worked examples.

### 4. Example 1: interrogative clause probability, direct conversations

<table>
<thead>
<tr>
<th></th>
<th>CL</th>
<th>CL(inter)</th>
<th>Words</th>
<th>$p$(inter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICE-GB</td>
<td>145,179</td>
<td>5,793</td>
<td>1,061,263</td>
<td>0.0399</td>
</tr>
<tr>
<td>spoken</td>
<td>90,422</td>
<td>5,050</td>
<td>637,682</td>
<td>0.0558</td>
</tr>
<tr>
<td>dialogue</td>
<td>57,161</td>
<td>4,686</td>
<td>376,689</td>
<td>0.0820</td>
</tr>
<tr>
<td>private</td>
<td>32,658</td>
<td>2,901</td>
<td>205,627</td>
<td>0.0888</td>
</tr>
<tr>
<td>direct conversations</td>
<td>29,503</td>
<td>2,617</td>
<td>185,208</td>
<td>0.0887</td>
</tr>
<tr>
<td>S1A-001</td>
<td>322</td>
<td>20</td>
<td>2,050</td>
<td>0.0621</td>
</tr>
<tr>
<td>S1A-002</td>
<td>328</td>
<td>19</td>
<td>2,055</td>
<td>0.0579</td>
</tr>
<tr>
<td>S1A-003</td>
<td>334</td>
<td>25</td>
<td>2,146</td>
<td>0.0749</td>
</tr>
<tr>
<td>S1A-090</td>
<td>326</td>
<td>44</td>
<td>1,968</td>
<td>0.1350</td>
</tr>
</tbody>
</table>

Table 1: Snippet of data table extracted from ICE-GB.

\(^1\) Our approach is worth comparing to that proposed by Singleton et al (1988) for random samples drawn from a small finite population. An observed value in a sample will tend to converge on the true population value as the sample becomes a greater proportion of the population. Singleton et al propose multiplying sample size $n$ by $v = 1 – n/N$, where $N$ is the finite population size. The effect of this multiplication is to increase $n$ and increase estimated confidence in results from the standard random-sample, infinite-population model.
Drawing frequency statistics from ICE-GB for interrogative clauses (denoted by ‘CL(inter)’) and all clauses (‘CL’), for each text in ICE-GB (approximately 2,000 words), we obtain a large hierarchical table, excerpted in Table 1. Here, “ICE-GB” represents the total across all of ICE-GB etc. There are 90 texts within the direct conversations category. This shows there are 29,503 clauses in 90 texts.

We assume that each text is approximately the same size and we make no assumptions about their further subdivision (separately-sampled subtexts, speakers), etc. We merely assume that these texts are randomly drawn from a population of comparable texts. Our approach is therefore conservative.

Consider the probability that a clause is interrogative ($p(\text{CL(inter)} | \text{CL})$, or $p(\text{inter})$ for short). For direct conversations,

$$\text{observed probability } p = p(\text{inter}) = \frac{f(\text{CL(inter)})}{f(\text{CL})} = 0.0887.$$  

If we make the standard assumption that the corpus is a random sample, we obtain the following standard deviation, variance and Wilson score intervals on this observation in the normal way (equations (1) and (2), see also Wallis 2013).

- **Number of cases** $n = f(\text{CL}) = 29,503$.
- **Standard deviation** $s = 0.001655$.
- **Wilson interval** $(w^−, w^+)$ = (0.0855, 0.0920) at a 95% error level.

However, this model assumes that the 29,503 clauses are drawn at random from the infinite population of English native-speaker direct conversations.

The question is: **to what extent are these measures of uncertainty an underestimate?**

First, we consider each text as a sample drawn from the population of similar-size texts. To what extent are interrogative clauses more common in some texts than others, and is this variation merely chance? A visualisation may help.

A frequency distribution of $p(\text{inter})$ across 90 texts may be obtained by quantisation and summation, i.e. rounding each probability into a series of discrete intervals, and summing the number of texts falling into this interval. This “frequency distribution by text” will tend to be Binomial.

Next, we compute a Normal approximation to this observed Binomial distribution. The mean of this distribution, $\bar{p}$, is simply the mean over the number of texts, so

$$\bar{p} = \sum_p \frac{p_i}{t} = 0.0890.$$

Employing equations (3) and (4) obtain

$$S_{ss} = \sqrt{\frac{\bar{p}(1-\bar{p})}{t}} = 0.0300,$$

and

Adapting variance for random-text sampling

Sean Wallis
Adapting variance for random-text sampling

\[
ss = \sqrt{\frac{\sum (p_i - \bar{p})^2}{(t-1)}} = 0.0395.
\]

The two Normal distributions for \(ss\), both centred on \(\bar{p}\), are shown in Figure 2, alongside the observed Binomial frequency distribution of texts.

The next step is to estimate the extent to which this distribution tends to ‘cluster’, i.e. the degree to which these interrogative clauses do not distribute randomly.

\[
\text{cluster-adjustment ratio } F_{ss} = \frac{S_{ss}^2}{s_{ss}^2} = 0.6935.
\]

\[\text{number of cases } n' = 20,460.\]

\[\text{standard deviation } s = 0.001988.\]

\[95\% \text{ Wilson interval } (w^-, w^+) = (0.0849, 0.0927).\]

The order of magnitude in this case is comparable to a suggestion we made on the corp.ling.stats blog in 2012: that dividing \(n\) by 2 would be a reasonable conservative estimate of the effect of compensating for within-text variance (Wallis 2012a).

4.1 Alternative method: Fitting

One concern may be that equation (4) does not appear to optimally match the observed discrete Binomial distribution (see Figure 2). An alternative method for estimating \(ss\) involves a computational search procedure.

By a process of minimising the sum of square differences between the observed distribution \(f(p)\) and the desired Gaussian probability function, \(Z\), we vary the standard deviation parameter \(ss\).

\[\text{error } e = \sum_{p=0,1} (Z(\bar{p}, ss, p) - f(p))^2.\]  \hspace{1cm} (6)

This method uses the Gaussian probability function \(Z(\bar{p}, s, p)\) with parameters mean \(\bar{p}\), standard deviation \(s\), and probability \(p\). By minimising \(e\), this obtains \(ss \approx 0.0474\).

Note that, apart from relying on a computationally intensive search procedure, this method also depends on a complex Gaussian function and the quantised \(f(p)\) distribution. In this case, the method of fitting obtains a greater estimate of spread than by direct computing using equation (4).

Using the fitted estimate for \(ss\) we obtain

\[
\text{cluster-adjustment ratio } F_{ss} = 0.4008.
\]

\[\text{number of cases } n' = 11,825.\]

\[\text{standard deviation } s = 0.002615.\]

\[95\% \text{ Wilson interval } (w^-, w^+) = (0.0837, 0.0940).\]

In this case, a scaling ratio of about 0.4 means that the variance for \(p(\text{inter})\) will be approximately 2.5 times greater than would be expected if cases were drawn randomly from the population rather than whole texts, and the standard deviation and Wilson error intervals are about 1.6 times greater (the standard deviation being the square root of the variance).

5. Example 2: Clauses per word, direct conversations

The next example utilises a word-based baseline, a baseline that is common in corpus linguistics for many purposes. This baseline obtains exposure probabilities, i.e. the probability that a reader or hearer will be exposed to a particular word, sequence or construction. It does not give us choice (or
utilisation) probabilities, unlike our first example. Word-based baselines also tend to be subject to between-genre sensitivity.

Suppose we wish to study the mean length of clauses, i.e. the number of words per clause. This variable is not a probability, but we can perform a simple mathematical trick to allow us to work with probabilities (Wallis 2012b). We take the reciprocal of the measure, i.e. the number of clauses per word, perform computations of variance, confidence intervals, statistical tests, etc., and finally invert this probability for citation and plotting purposes. Another way of expressing the number of clauses per word is that it is the probability that a random word is the first word in a clause.

Using the ICE-GB direct conversation data we obtain the following

\[
\begin{align*}
\text{observed probability } p &= p(\text{CL}) = f(\text{CL})/f(\text{word}) = 0.159297. \\
\text{number of cases } n &= f(\text{word}) = 185,208. \\
\text{standard deviation } s &= 0.00850. \\
\text{95\% Wilson interval } (\tilde{w}^-, \tilde{w}^+) &= (0.15919, 0.15937).
\end{align*}
\]

This is a small interval due to the large number of words, \(n\), obtained. Next, we examine the distribution of texts. Does the rate of clauses per text vary more than would be expected by chance?

\[
\begin{align*}
\text{distribution mean } \tilde{p} &= 0.159277. \\
\text{predicted standard deviation } S_{ss} &= 0.038573. \\
\text{observed standard deviation } s_{ss} &= 0.013781 \text{ (by computation, equation (4)).}
\end{align*}
\]

The scaling ratio is

\[
\text{cluster-adjustment ratio } F_{ss} = S_{ss}^2/s_{ss}^2 = 7.8340.
\]

This result is, initially at least, rather surprising. So far we have assumed that the predicted standard deviation (calculated by assuming subsamples are proper random samples), would be smaller than the observed standard deviation, based on the distribution of the subsamples themselves. But in this case the predicted standard deviation is nearly three times greater than the observed! A slight increase might be explained by rounding errors. But this is a substantial increase. What is going on?

The fitting method for \(s_{ss}\) obtains almost exactly the same result in this case. This is not an artefact of an erroneous calculation of the observed standard deviation. The explanation must lie elsewhere.

The Binomial model for a variable \(p\) assumes that \(p\) is free to vary from 0 to 1. Logically, in our example, this would mean that every word in the corpus could be the first word in a clause. But this is simply
impossible in practice. However many times we sample clauses, \( \max(p) \) will be much less than 1.

If the range of \( p \) is compressed, then the Binomial model, and approximations to it (1), necessarily overestimate the variance of \( p \), and the degree of overestimation can be substantial (Wallis 2012c). Many linguistic variables, including word-based probabilities, do not maximise at 1. It is unrealistic to assume that an entire corpus would consist of a given word, structure, etc., yet many methods make precisely this freedom-to-vary assumption. A free choice between lexical or grammatical alternates may range from 0 to 1, but the number of clauses per word is not such a variable.

By examining the frequency distribution of subsample mean probabilities, we observe the actual behaviour of \( p \) and how it distributes, and obtain \( s_{ss} \). As a side-effect of our method, therefore, we appear to be able to compensate for this kind of overestimation of intervals. We are entitled to increase \( n \) by \( F_{ss} \) and reduce the confidence intervals still further.

\[
\text{number of cases } n' = 1,450,933. \\
\text{standard deviation } s = 0.000304. \\
95\% \text{ Wilson interval } (w^-, w^+) = (0.159258, 0.159299).
\]

The argument is that we are drawing more information from the sample, and thus we are entitled to improve the precision of our tests, confidence intervals, etc. This increases the absolute variance.

6. Uneven-size subsamples

Note that the subsample distribution mean, \( \bar{p} \) (averaged over subsamples) and the sample-wide mean, \( p \) (averaged over instances) are not necessarily the same. The reason they differ is that subsamples can vary in size. However, equation (4) assumes that all subsamples are the same size.

In the previous illustrative examples, subsample sizes were fairly consistent, so this is not really an issue. However, for robustness, we should allow for cases where source texts may vary in size (i.e. texts are permitted to vary in word length). In ICE-GB, for example, some texts are composed of shorter independent subtexts. Moreover, even if the number of words per text was exactly constant, for any given research question the number of instances abstracted from the text, \( n_i \), may vary. In examining the tendency for a clause to be interrogative, we should take heed of the fact that the number of clauses per word also varies between texts (Figure 3).

For this reason, a more accurate estimate of between-subsample variance may be obtained by taking \( p = \bar{p} \) and using the following probabilistically-weighted formula in place of equation (4).

\[
s_{ss} = \sqrt{\frac{t}{(t-1)\sum pr_i(p_i - \bar{p})^2}} = \sqrt{\frac{t}{n(t-1)\sum n_i(p_i - \bar{p})^2}}, \tag{7}
\]

where prior probability \( pr_i = n_i/n \). This is the probability that were you to draw a random instance from the sample, the instance would be found in subsample \( i \). The number of texts, \( t \), in equations (3) and (7) is limited to cases where \( n_i > 0 \).

Using equation (7) in place of (4), we obtain \( s_{ss} = 0.0397 \) (Example 1) and \( 0.013856 \) (Example 2). In these examples, standard deviations increase by \( \approx 0.5\% \), which is negligible.

7. Example 3: Interrogative clause probability, all ICE-GB data

So far we have considered the effect of measuring the between-subsample variation for a variable within a single ICE-GB text category. Finally, we consider a situation of sampling across many text genres for a variable known to vary substantially across them.
ICE-GB contains both spoken and written English. The probability of selecting an interrogative clause varies widely between different text categories. We can expect that question-clauses in written texts and monologues are likely to be much less frequent than in direct conversations. But if our method is to be robust and scalable, we need to consider research questions like these.

Using all 500 ICE-GB text categories we obtain the following:

\[ \text{observed probability } p = p(\text{CL}) = \frac{f(\text{CL})}{f(\text{word})} = 0.0399. \]
\[ \text{number of cases } n = f(\text{CL}) = 145,179. \]
\[ \text{standard deviation } s = 0.000514. \]
\[ 95\% \text{ Wilson interval } (w^-, w^+) = (0.038908, 0.040922). \]

Employing the probabilistically-weighted method using \( p \) obtains:

\[ \text{predicted standard deviation } S_{ss} = 0.008753. \]
\[ \text{observed standard deviation } s_{ss} = 0.044462 \text{ (by computation, equation (7)).} \]
\[ \text{cluster-adjustment ratio } F_{ss} = \frac{S_{ss}^2}{s_{ss}^2} = 0.038758. \]

The variation of \( p \) between different texts is much larger than the variation expected by the random sampling assumption. See Figure 4. This result entails a reduction of \( n \) by approximately \( \frac{1}{26} \). Following rescaling, we obtain

\[ \text{observed probability } p = p(\text{CL}) = 0.0399. \]
\[ \text{number of cases } n' = 5,627. \]
\[ \text{standard deviation } s = 0.003061. \]
\[ 95\% \text{ Wilson interval } (w^-, w^+) = (0.035094, 0.045338). \]

Although we are rescaling the interval by a factor of \( \frac{1}{26} \), because the initial interval was tiny, the new interval is still small. If you compare the resulting sample size \( n \) with Example 1 (direct conversation data only), it is worth noting that, despite the much more diverse sample, the standard deviation of \( p \) has increased by 53\% (by computed \( s_{ss}, 0.001988 \)).

Whereas a method based on ratio of variances should be sound and robust, there remains one potential concern. Figure 4 illustrates a known problem with the Normal approximation to the Binomial. As \( p \) approaches 0 or 1, the Binomial tends to an assymmetric Poisson distribution rather than approximating to the Normal. In such a distribution the mean (the average value) is not the same as the mode (the most frequent value). A symmetric Gaussian curve cannot converge on this frequency distribution, and the lower tail of the distribution with standard deviation \( s_{ss} \) is severely cropped.

Figure 4: Frequency distribution for \( p(\text{inter}) \), all ICE-GB texts, with Gaussian curves for predicted and observed measures of standard deviation.
8. Example 4: Rate of transitive complement addition

As a final example, we will take a grammatical variable and apply it to the text categories in ICE-GB. A typical variable is illustrated by the two Fuzzy Tree Fragments (FTFs, Nelson et al. 2002) in Figure 5. Figure 5a retrieves the baseline set (cases of non-passive clauses), whereas Figure 5b retrieves the subset where the verb phrase is followed by a non-passive, infinitive transitive complement clause containing a subject. The variable we wish to evaluate in this case, \( p \), may be expressed as the tendency to add such transitive complements to the upper, ‘host’, clause.

Figure 6 plots the distribution of \( p \) with Wilson intervals across ICE-GB genre categories. The thin ‘I’-shaped error bars represent the conventional Wilson score interval for \( p \), assuming random sampling. The thicker error bars represent the adjusted Wilson interval obtained using the probabilistically-weighted method of equation (7). These results are tabulated in Table 2.

The figure reinforces observations we made earlier. Within a single text type, such as broadcast interviews, \( p \) has a compressed range and cannot plausibly approach 1. (Note that mean \( p \) does not exceed 0.03 in any genre.) The observed between-text distribution is smaller than that predicted by equation (3), and, armed with this information, we are able to reduce the 95% Wilson score interval for \( p \). This degree of compression (i.e., the plausible value of max(\( p \))) may also differ by text genre.

However, the reduction due to range-compression is offset by a countervailing tendency: pooling genres increases the variance of \( p \). The distribution of texts across the entire corpus consists of the sum of the spoken and written distributions (means 0.0091 and 0.0137 respectively), and so on.
Table 2: Hierarchical table of results for following a verb phrase with a transitive complement, with Wilson intervals before and after adjustment, adjustment ratio $F_{\text{adj}}$, and other variables.

The Wilson interval for the mean $p$ averaged over all of ICE-GB approximately doubles in width ($F_{\text{adj}} = 0.2504$), and the intervals for spoken, dialogue, private, written and printed (marked in bold in Figure 6) also expand, albeit to lesser extents. Other intervals contract ($F_{\text{adj}} > 1$), tending to generate a more consistent set of intervals over all text categories.
9. Conclusions

In this paper we derived and demonstrated a method for adjusting the estimated variance of a sample composed of contiguous subsamples which is highly generalisable to other methods. Where we find that subsamples are more widely spread than would be expected by chance random sampling, our approach simply reduces the weight of evidence, represented by sample size \( n \), supporting an observation \( p \).

As a by-product of the procedure of examining actual distributions of \( p \), we also compensate for another issue, namely that many ‘probabilistic’ variables conventionally measured by linguists are not plausibly able to reach 100% saturation. Without this compensation, standard methods can vastly overestimate the range of values for \( p \): in our final example we found confidence intervals that were more than 5 times oversized. Correcting for this problem, which is mostly not discussed in the literature, increases the statistical sensitivity of tests substantially.

This paper is based on sample-wide estimation. Unlike the case interaction approach, it is not built up from individual instances. We rely on less information than may be obtained by such models. On the other hand, we rely on more information than a Mann-Whitney approach. Instead of rank positions, we use the actual scores per text, and we can make claims about our observation \( p \). We extended our method to subsamples of uneven sizes, which is not trivial to address with a rank order statistic. Our approach does not dispense with the Binomial model, but adjusts it, so it suffers no loss of generality. Thus, for example, it is a simple matter to compare any pair of adjusted Wilson intervals in Figure 6 with Newcombe’s method (Wallis 2013).

Our approach is likely to be more conservative (“weaker”, or a less powerful test) than an optimal case interaction model, and is also less conservative (“stronger”) than the Mann-Whitney U test.

One direction of future and ongoing work remains the development of case interaction models. As we noted earlier, these models have a number of potential desirable characteristics for linguistic study in their own right. Our method is premised on observing the distributional consequences of many interactions, rather than examining each interaction in turn. To draw a medical analogy, it is the difference between plotting the epidemiological consequences of contagion rather than modelling the process of contagion itself. The method described in the present paper can clearly be employed to parameterise such case interaction models. It gives us an order of magnitude estimate of the maximum extent of the sum of between-case interdependence measures.

References


Adapting variance for random-text sampling - 12 - Sean Wallis