

# CHAPTER 1. THE THEORY OF HEDONIC MARKETS

## *a. Introduction*

One of the most familiar models in economics is that of price determination in the market. The market for a particular good consists of a large number of consumers whose demand for the good is met by the production of a large number of firms. The market mechanism works to reconcile the needs of consumers and firms by establishing the price at which aggregate demand is equal to aggregate supply and the market clears. At this price the market is said to be in equilibrium since there is no excess demand for the good and firms cannot increase their profits by changing their production of the good.

For many goods, however, this simple model is inadequate. For example, the simple model predicts that once in equilibrium the market will determine one price for the good. However, in a market such as that for housing we observe different properties commanding different prices. Indeed, housing is an example of what is called a *differentiated good*. Such goods consist of a diversity of products that, while differing in a variety of characteristics, are so closely related in consumers' minds that they are considered as being one commodity. Many other goods, including breakfast cereals, cars and beach holidays also fit this description.

Though the simple model does not adequately explain the workings of markets in differentiated goods, it would appear that a similar market mechanism is in operation. Market forces determine that different varieties of the product command different prices and that these prices depend on the individual products' exact characteristics. For example, properties that have more bedrooms will tend to command a higher price in the market than properties that have fewer bedrooms. Furthermore, the set of prices in the market would appear to define a competitive equilibrium. That is, in general, the market will settle on a set of prices for the numerous varieties of the differentiated good that reconcile supply with demand and clear the market.

In a seminal paper, Rosen (1974) proposed a model of market behaviour that described the workings of markets for differentiated goods. The model that Rosen presented provides the theoretical underpinnings for hedonic valuation and will provide the subject matter of this first section.

## *b. The Property Market: The Differentiated Good*

As a consequence of the fundamentally spatial nature of property, property markets are themselves defined spatially. We shall assume that at any point in time, all of the properties in one urban area represent the products in the property market. The *households* wishing to live in these properties represent the consumers in this market and the *landlords* that own the properties represent the producers in this market<sup>1</sup>.

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<sup>1</sup> Within this basic hedonic price model we do not consider the possibility of migration between towns.

Clearly the set of properties in the market represent a differentiated good. We could describe any particular property by the qualities or characteristics of its structure, environs and location. A succinct means of denoting this is as a vector of values; effectively a list of the different quantities of each characteristic of the property. In general, therefore, any house could be described by the vector,

$$\mathbf{z} = (z_1, z_2, \dots, z_K), \quad (1)$$

where  $z_i$  ( $i = 1$  to  $K$ ) is the level or amount of any one of the many characteristics describing a property. Indeed, the vector  $\mathbf{z}$  completely describes the services provided by the property to a household.

For the sake of simplicity let us assume that the  $z_i$  are measured in such a way that we can consider them as “goods” as opposed to “bads”. For example, one of the characteristics of a property will be its exposure to road noise. Rather than measuring this as the level of “noise”, we can simply invert the scale and measure it as the level of “peace and quiet”.

Further, let us assume that the set of properties in the market is fixed. That is we assume that in the short-run no new properties are built. That is not to say that the characteristics of properties do not change. A landlord maintains the quality of the property by constant renovation and maintenance. Alternatively the landlord can improve the quality of the property through investment. Building an extension, converting a loft or basement, installing double-glazing or central heating, improving the quality of the décor, indeed carrying out any number of alterations and improvements can increase the values of certain of the characteristics of the property. On the other hand, disinvesting, that is failing to maintain and renovate the property, will lead to the quality of certain of its characteristics declining. Of course, certain characteristics of the property cannot be influenced by the actions of the landlord. Most notably the landlord has little influence over the characteristics of the property that are location specific such as its proximity to places of work or to local amenities, or the property’s exposure to noise and air pollution.

When households select a particular property in a particular location they are selecting a particular set of values for each of the  $z_i$ . We can imagine this market for properties as being one in which the consumers consider a variety of somewhat dissimilar products which differ from each other in a number of characteristics including, amongst many characteristics, number of rooms, size of garden, distance to shops and environmental characteristics such as levels of pollution or noise. Using an analogy of Freeman (1993 p 371), “it is as if the urban area were one huge supermarket offering a wide selection of varieties. Of course, the individuals cannot move their shopping carts through this supermarket. Rather, their selections of residential locations fix for them the whole bundle of housing services. It is much as if shoppers were forced to make their choices from an array of already filled shopping carts. Individuals can increase the quantity of any characteristic by finding an alternative location alike in all other aspects but offering more of the desired characteristic.”

### ***c. The Property Market: The Hedonic Price Function***

The price of any one of these ‘shopping carts’ will be determined by the particular combination of characteristics it displays. Naturally we would expect properties possessing larger quantities of good qualities to command higher prices and those with larger quantities of bad qualities to command lower prices. Again we can use a succinct piece of notation to illustrate this point;

$$P = P(\mathbf{z}) \quad (2)$$

Which can be read as; the price of a property,  $P$ , is a function of the vector of values,  $\mathbf{z}$ , describing its characteristics. This function is known as the *hedonic price function*; ‘hedonic’ because it is determined by the different qualities of the differentiated good and the ‘pleasure’ (in economic terms utility) these would bring to the purchaser.

In the property market this price is the rental that a household pays to the landlord. In effect, every household in the urban area is purchasing the flow of services derived from the characteristics of the property per period of time. To clarify  $P(\mathbf{z})$  is the per period payment made by a household to a landlord for the use of a property over that period.

Of course, many households own their own homes. In this case we treat homeowners as landlords that rent from themselves. If markets are operating perfectly, and generally we assume that they are, then the price at which the household purchases the property will be the discounted sum of all the future per period rents from that property according to;

$$\text{Purchase Price} = \sum_{t=1}^T \frac{P(\mathbf{z})}{(1+d)^t} \quad (3)$$

where  $t$  indexes each time period,  $T$  is the expected life of the property and  $d$  is the discount rate. Naturally, using Equation (3) it’s a relatively easy task to translate purchase prices into per period rentals.

Whilst the analogy of the ‘shopping cart’ is instructive in clarifying the nature of a differentiated good it is somewhat misleading in the determination of the price of that good. In particular, consider two shopping carts containing identical bundles of goods. Naturally having identical characteristics these two shopping carts would command the same price. Now imagine that a loaf of bread was added to one of the carts. The price paid for this cart would increase by the price of a loaf of bread. If another loaf were added to the same cart its price would again increase by the same amount. In general, loaves of bread don’t become cheaper or more expensive the more you purchase. Indeed, we can state that when adding a particular good (i.e. characteristic) to a shopping cart the increase in its price will be the same no matter what combination or quantity of goods are already contained in that cart. In economic terms we would say that the marginal price of each characteristic is constant.

The market forces that ensure that marginal prices are constant are known in economic terms as “arbitrage”. Returning to our example, since buying two carts both containing one loaf of bread confers the same benefit on a consumer as purchasing one cart containing two loaves of bread, arbitrage activity ensures that the marginal price of bread is constant. To expand, if the cart with two loaves were more expensive than twice the price of the carts containing one loaf each, then a rational shopper would always chose to push the two single-loafed carts through the check-out. The result would be a lack of demand for two-loafed carts and excess demand for one-loafed carts. Market forces would work so as to bring the price of two-loafed carts down and increase the price of one-loafed carts. Only when the price of the former were twice the price of the latter would the market reach an equilibrium. In this equilibrium, arbitrage activity has worked to ensure that the marginal price of extra loaves in a shopping cart is constant.

However, the same sort of activity is frequently impossible in the purchase of a truly differentiated good such as a property. This feature of hedonic markets results from the fact that households are unable to “repackage” the differentiated goods. In other words, households cannot break up the differentiated good into its constituent parts and enjoy the benefits of each characteristic separate from the whole. For example, talking in terms of just one characteristic, two houses with one bedroom are not equivalent to one house with two bedrooms since a household cannot live in both properties simultaneously. Similarly, renting a property with four bedrooms for half a year and a property with two bedrooms for the other half is not the same as renting a three-bedroom house all year round. Since these types of arbitrage activity are precluded in the housing market, market forces do not work to ensure that the marginal price of bedrooms is constant.

This observation leads to two interesting insights.

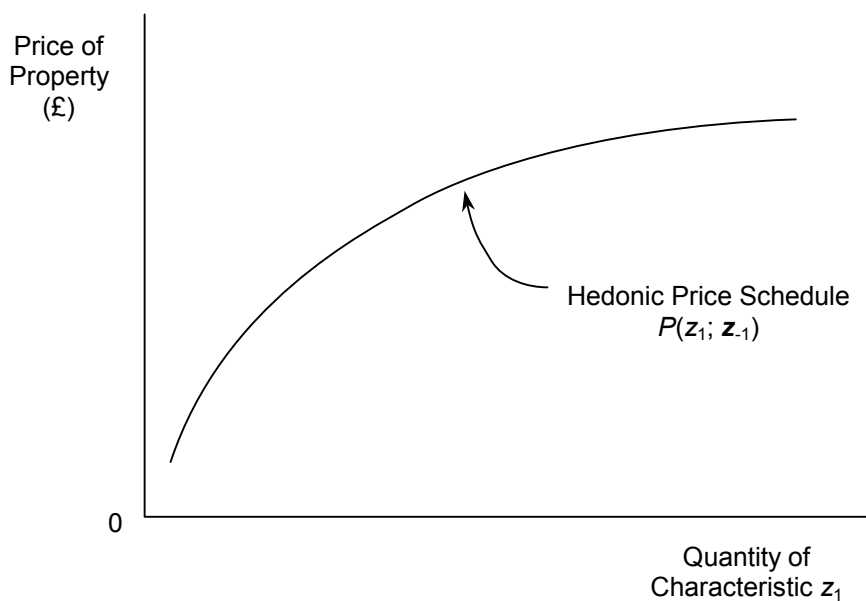
- *Marginal prices may not be constant.* To illustrate, imagine a number of properties that are identical in all characteristics except one. Within the housing market we may find that the extra paid for properties with additional units of this particular characteristic is not constant. Indeed, more typically, the additional amount paid for properties enjoying increasingly higher quantities of the characteristic (in effect the price of that characteristic) declines as the total level of that characteristic increases.
- *The price of one characteristic may depend on the quantity of another.* As an example, a house with a garden is more desirable than a house without. Further, if the aspect of the house is north-south, having a garden may be even more desirable since it will enjoy longer exposure to the sun. Now consider the extra paid for a north-south aspect, effectively the ‘price’ of north-south aspect. Without a garden, north-south aspect may be somewhat desirable, but households are unlikely to pay a great deal more for a property with this characteristic compared to an identical property with east-west aspect. For properties with a garden, on the other hand, aspect may be a much more important consideration. It would not be surprising that the price of north-south aspect will depend on whether a property has a garden or not.

These two observations will be important in empirical applications that attempt to estimate the hedonic price function from market data.

To illustrate the hedonic price function, consider the illustration in Figure 1. Plotted on the vertical axis is the price (rental per unit time) of property. Along the horizontal axis is

quantity of a particular housing characteristic labelled  $z_1$ . For illustrative purposes let us assume that this characteristic is the size of the property's garden. Further, let us introduce some new notation,  $z_{-1}$ , which is the vector containing the levels of all property characteristics barring  $z_1$ . Notice that in the hedonic price function in Figure 1,  $z_{-1}$  comes after a semicolon. This indicates that these other characteristics are held constant at some given level whilst the focus characteristic, size of garden (i.e.  $z_1$ ), changes. Consequently, in this example we are not considering the interaction of different characteristics of the property.

**Figure 1: The Hedonic Price Schedule for characteristic  $z_1$**



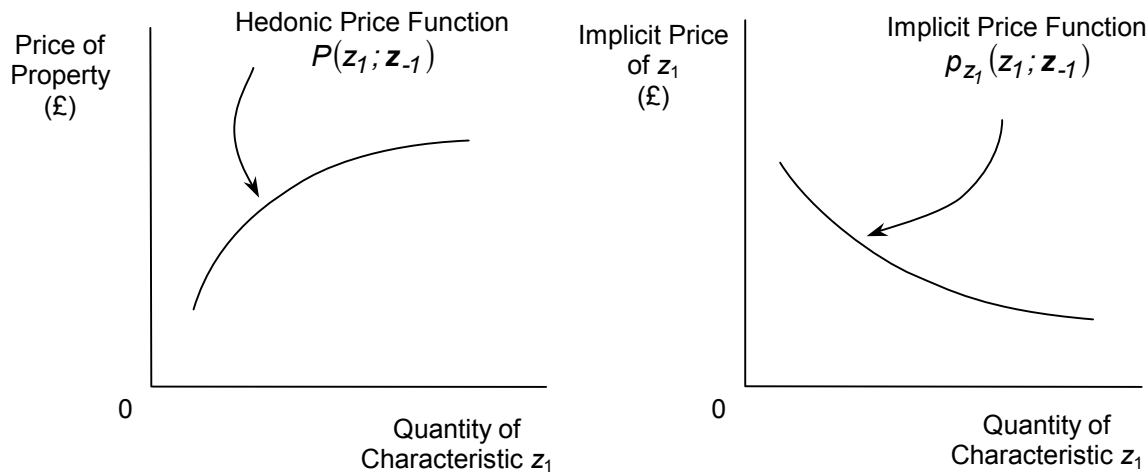
In this hypothetical case, the hedonic price function rises from left to right implying that the bigger a property's garden the higher the price that property commands in the market. Notice also that the marginal price of extra garden space is not constant. The slope of the curve becomes progressively flatter and the incremental increase in a property's market price resulting from its possessing a bigger garden declines as gardens get progressively larger. This sort of relationship reflects a form of satiation; having a few square metres of garden will add considerably to the price of a house when compared to a house with no garden at all, whilst a few extra square metres will make a negligible difference between the selling prices of two houses which already boast football pitch-sized gardens.

Of course the relationship won't be identical to that graphed for every type of characteristic. For example, we might find that for another characteristic, say floor space, plotting house price against the quantity of the characteristic (again holding all other characteristics constant) results in a straight line rising from left to right. A straight line relationship suggests that there is no satiation and that the price commanded by a property increases uniformly in relation to the quantity of the characteristic that it possesses. In other words the characteristic would possess a constant marginal price.

It may be easier to illustrate the idea of non-constant marginal prices through actually plotting the additional amount that must be paid by any household to move to a bundle with a higher level of that characteristic, other things being equal. This is illustrated in the right hand panel of Figure 2.

This new function is known as the *implicit price function*; *implicit* because the marginal price of a characteristic is revealed to us indirectly through the amounts households are prepared to pay for the whole property of which the particular characteristic is only a part. From Figure 2, we can see that at first the hedonic price function rises steeply so that the implicit price of the characteristic (the extra amount paid to acquire a house with more of characteristic  $z_1$ ) is also high. At higher levels of  $z_1$  the hedonic price function is flatter so that the implicit price of the characteristic is also low.

**Figure 2: The Hedonic Price and the Implicit Price Schedules for characteristic  $z_1$**



Mathematically, the implicit price is derived as the partial derivative of the hedonic price function (Equation 2) with respect to one of its arguments,  $z_i$ , according to:

$$p_{z_i}(z_i, z_{-i}) = \frac{\partial P(z)}{\partial z_i} \quad (4)$$

Again  $p_{z_i}(z_i, z_{-i})$ , the marginal price function of characteristic  $z_i$ , does not have to be a constant. We have dwelt on the subject of non-constant marginal prices for characteristics since as we, shall see in a later section they are the source of much of the complications that confound the empirical estimation of welfare measures using hedonic analysis.

#### ***d. The Property Market: Household Choice***

Let us take as a fact that the hedonic price function,  $P(\mathbf{z})$ , emerges from the interaction of households (demanders) and landlords (suppliers) and represents a market clearing equilibrium. We shall return to the mechanism by which this equilibrium is derived, but for now we shall focus on how households facing such a hedonic price schedule determine their optimal residential location.

The model that Rosen developed to explain these decisions is based on a number of assumptions. Amongst these, some of the most important are that;

- Each individual household in the market is a price taker; they make their choice of location based on the hedonic price schedule they observe in the market and cannot influence this schedule through their actions. This point has been made by several authors (McConnell and Phipps, 1987; Palmquist, 1991) and allows us to ignore the supply side of the market in modelling households' residential decisions. Given the size of the urban property markets in which hedonic pricing techniques are usually applied, such an assumption would appear reasonable.
- Each household only purchases or rents one property<sup>2</sup>. If households purchase a second home, say a holiday home, then this should be considered as a separate good being purchased in a separate hedonic market. Again, this assumption is, in general, readily defensible.

Given these assumptions, Rosen sets out a model in which households choose their residential location so as to get the maximum flow of benefits or, in economic terms, utility from the property. To do this, it is assumed that households in the market have well-defined preferences over all goods and that these preferences can be represented by the utility function<sup>3</sup>;

$$U(\mathbf{z}, x; \mathbf{s}) \tag{5}$$

As should be familiar to most readers, the utility function gives the utility that a household enjoys per period of time, given the levels of the arguments contained in the brackets. In this case, there are three arguments;

- $\mathbf{z}$  which represents the levels of the different characteristics of property that a household could purchase or rent. Naturally the utility that a household enjoys each period of time will depend on the qualities of the property in which they choose to live.

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<sup>2</sup> Further, if households also act as landlords to other households, then their decisions concerning their own choice of residential location are assumed to be independent of their decisions concerning these other properties.

<sup>3</sup> The utility function is assumed to be identical for all households in the market. However, the actual utility derived from a certain property with characteristics,  $\mathbf{z}$ , and a certain quantity of other goods,  $x$ , will depend upon the characteristics of the household,  $\mathbf{s}$ .

- $x$  which represents all other goods outside the property market. As a matter of convenience we standardise  $x$  to have a unit price, such that, we are effectively representing all other goods by a quantity of money<sup>4</sup>. Again the greater this quantity of money to spend on other goods, the more utility a household will enjoy per period time.
- $s$  which represents the characteristics of the household themselves. Clearly, the quantity of utility a household enjoys from any of the other arguments will depend on their own characteristics. For example, having a swimming pool in the back garden will confer little benefit on a household of non-swimmers. Again, notice that this argument is placed after a semicolon in the utility function. This indicates that the level of utility that a household gets from any of the arguments over which it has a choice (i.e.  $z$  and  $x$ ) will be dependent upon (or conditional upon) their own characteristics.

For the purposes of developing a model of behaviour we don't specify the exact form of the utility function<sup>5</sup>. In other words we don't need to state that the quantity of utility that a household experiences will be, for example, 2 times the number of bedrooms plus 4 times the size of the garden, plus -.02 times the number of decibels of road traffic, plus ... etc. For our purposes, we can continue just assuming there is such a function and that it is the same for each household conditional upon their characteristics<sup>6</sup>.

Households choose levels of  $z$  and  $x$  to maximise  $U(z, x; s)$  subject to the constraints imposed upon them by their budget. Since the price of  $x$  is taken as unity and the price of a property with characteristics  $z$  is given by the hedonic price function  $P(z)$ , we can represent the budget constraint as;

$$y = x + P(z) \tag{6}$$

where  $y$  is household income per period.

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<sup>4</sup> In the economics literature  $x$  is frequently referred to as a numeraire, a composite good or a Hicksian bundle.

<sup>5</sup> We do however assume that the utility function is strictly increasing in the arguments  $x$  and  $z$ . That is we assume that the household prefers more attributes to less attributes (remember we already assumed that all attributes were measured as goods rather than bads) and more money to spend on other goods than less. Further, for mathematical simplicity, we assume that the utility function is strictly quasiconcave and twice continuously differentiable.

<sup>6</sup> Some authors (e.g. Epple, 1984) do in fact impose some specific functional form on the utility function and use this as a means of investigating prices and choices in hedonic markets. We do not consider such models here.



As with the standard consumer choice problem, we can use Equations (5) and (6) to set up the Lagrangian Function;

$$L = U(\mathbf{z}, x; \mathbf{s}) + \lambda(y - x - P(\mathbf{z})) \quad (7)$$

Maximising this with respect to  $x$ ,  $\mathbf{z}$  and the Lagrange multiplier  $\lambda$  gives rise to the first order conditions;

$$\frac{\partial L}{\partial z_i} = U_{z_i} - \lambda P_{z_i} = 0 \quad (i = 1 \text{ to } K) \quad (8)$$

$$\frac{\partial L}{\partial x} = U_x - \lambda = 0 \quad (9)$$

$$\frac{\partial L}{\partial \lambda} = y - x - P(\mathbf{z}) = 0 \quad (10)$$

Where  $U_{z_i}$  is the partial derivative of the utility function with respect to property characteristic  $z_i$ . This represents the extra utility that comes from choosing a property with one extra unit of characteristic  $z_i$ , all else being equal.

$U_x$  is the partial derivative of the utility function with respect to the composite good. Since  $x$  is constructed so as to represent money to be spent on other goods,  $U_x$  can be interpreted as the extra utility that comes from an extra unit of money, all else being equal.

$P_{z_i}$  is the partial derivative of the hedonic price function with respect to property characteristic  $z_i$ . Of course this is simply the implicit price function for characteristic  $z_i$  as presented in Equation (4). Indeed,  $P_{z_i} = p_{z_i}(z_i, \mathbf{z}_{-i})$ .

Equations (8), (9) and (10) represent the conditions that define the household's optimal choice of residential location. That is, given the constraint of their budget, the flow of utility that the household enjoys will be maximised by choosing a property whose characteristics simultaneously satisfy the conditions laid out in Equations (8), (9) and (10).

In their present form these conditions provide us with little insight into the household's choice behaviour. However, if we rearrange Equations (8) and (9) and divide one by the other (thereby eliminating the Lagrange multiplier) we reveal that one of the conditions for optimal choice is given by the expression;

$$\frac{U_{z_i}}{U_x} = p_{z_i}(z_i, z_{-i}) \quad (11)$$

To illustrate the condition laid out in Equation (11) Rosen defined a function that he termed the *bid function*, whose slope is given by the ratio of marginal utilities,  $U_{z_i}/U_x$ . Most students of economics will be familiar with terms involving ratios of marginal utilities. More usually we would expect to see the ratio of two marginal utilities preceded by a negative sign. In such a case, the expression would represent a *marginal rate of substitution*, e.g.  $-U_{z_i}/U_x$ ; the quantity of one good that a household is willing to give up in order to obtain one more unit of another good such that their overall well-being does not change. In the same way that  $U_{z_i}/U_x$  defines the slope of Rosen's bid function, the marginal rate of substitution defines the slope of an indifference curve. In hedonic analysis there is a simple correspondence between the indifference curve and the bid function that goes some way in clarifying the nature of the latter.

Let us spend a little time considering indifference curves. In mathematical terms, the indifference curve is implicitly defined as;

$$U(z, x; s) = u \quad (12)$$

Where  $u$  is any specified level of utility. Thus, the indifference curve depicts combinations of  $x$  and  $z$  that confer the same level of well-being or utility on the household. Indeed, solving Equation (12) for  $x$  would give us a general expression for an indifference curve that we can denote;

$$x(z; s, u) \quad (13)$$

Written in this form, the indifference curve tells us what quantity of money to spend on other goods,  $x$ , would allow a household with characteristics,  $s$ , to enjoy the level of utility  $u$ , given they lived in a property with characteristics  $z$ .

The left hand panel of Figure 3 shows a set of indifference curves between  $x$  (the quantity of money to spend on other goods) and  $z_1$  (one of the attributes of a property)<sup>7</sup>. Most readers will be familiar with this diagram. Each indifference curve depicts combinations of  $x$  and  $z_1$  that confer the same level of well-being or utility on the household.

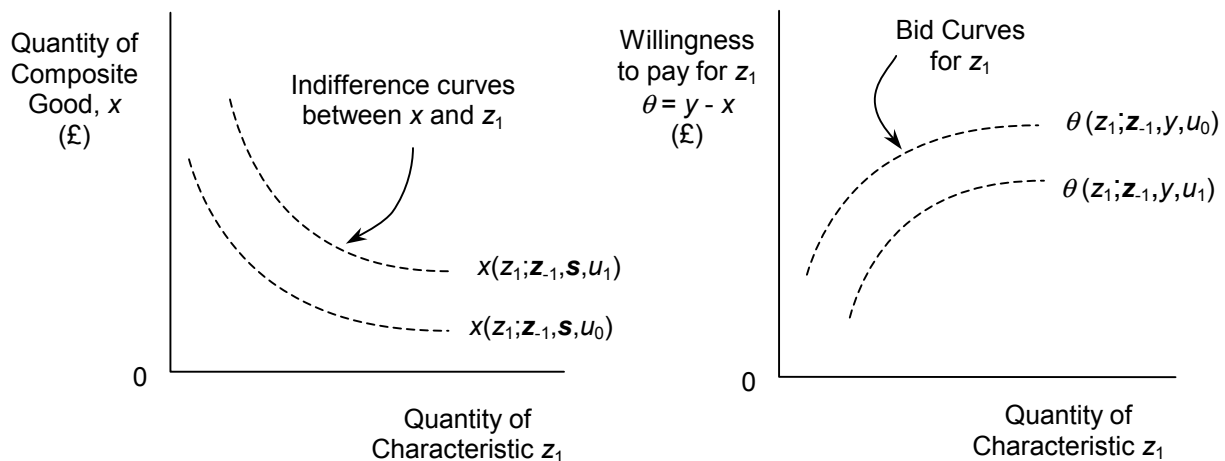
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<sup>7</sup> For diagrammatic exposition, it is necessary to present indifference curves in terms of only one property attribute. The assumption in Figure 3 is that all other attributes are held at some constant level. In reality, the indifference 'curve' would be a multidimensional indifference surface plotting combinations of  $x$  and quantities of each of the attributes in  $z$  between which the household is indifferent.

Notice that when levels of attribute  $z_1$  are low, the household is willing to give up quite a lot of their money to spend on other goods in order to acquire a property with greater levels of the attribute. Conversely, when levels of attribute  $z_1$  are high the household is only prepared to sacrifice a small amount of money to spend on other goods in order to acquire more of the attribute<sup>8</sup>. The slope of the indifference curve gives the rate at which households are prepared to give up money for other goods in order to acquire more of the housing attribute whilst not changing their overall well-being. As discussed previously, the slope of the indifference curve is the marginal rate of substitution between  $x$  and  $z_1$ ,  $-U_{z_1}/U_x$ .

Any combination of  $x$  and  $z_1$  that lies above and to the right of an indifference curve provides the household with more money to spend on other goods and/or more of the housing attribute. By definition they must gain more utility from such a bundle than from any bundle lying on the indifference curve. Consequently the indifference curve for  $u_1$  must represent bundles of  $x$  and  $z_1$  between which the household is indifferent but which all confer more utility than bundles lying along the indifference curve for  $u_0$ .

**Figure 3: Indifference Curves and the Bid Function**



So far we have not considered the fact that the household is constrained in their choices of  $x$  and  $z$  by their limited budget,  $y$ . Money spent on other things is money that cannot be spent on housing attributes. Let us define, therefore, an amount that we shall call a *bid* as;

$$\theta = y - x \tag{14}$$

<sup>8</sup> This “classic” shape for the indifference curve stems from our assumptions concerning the utility function described in footnote 14.

That is the bid,  $\theta$ , represents the total amount a household could pay for a property given that they spent  $x$  on other goods<sup>9</sup>. Clearly the relationship between the bid,  $\theta$ , and the amount spent on other goods,  $x$ , is very simple; as one goes up by a certain amount the other falls by the same quantity<sup>10</sup>. Indeed, using Equation (14) we could redefine the indifference relationships of Equation (13) in terms of bids rather than money spent on other goods. Replacing Equation (13) in Equation (14) gives;

$$\begin{aligned}\theta &= y - x(\mathbf{z}; \mathbf{s}, u) \\ &= \theta(\mathbf{z}; y, \mathbf{s}, u)\end{aligned}\tag{15}$$

The bids defined by Equation (15) are a special type of bid. They are bids for a property with characteristics  $\mathbf{z}$  that result in the level of utility  $u$ . Indeed, Equation (15) defines Rosen's *bid function*.

In words, the bid function depicts the maximum amount that a household would pay for a property with attributes  $\mathbf{z}$  such that they could achieve the given level of utility,  $u$ , with their income,  $y$ . Notice that increases in income translate directly (i.e. pound for pound) into increases in the bid function.

The bid function can be illustrated as *bid curves* as depicted in the right hand panel of Figure 3. In constructing these bid curves, all that has been done, in effect, is to flip the vertical axis. Bid curves still define indifference relationships. They depict combinations of property attributes,  $\mathbf{z}$ , and payments for those attributes,  $\theta$ , between which the household is indifferent. Accordingly, all bid/attribute-quantity combinations on a particular bid curve provide the household with the same level of overall utility. Combinations of housing attributes and bids lying below and to the right of a particular bid curve represent bundles providing more attributes and/or lower payments. Clearly the household would gain more utility from such a bundle. Consequently the lower bid curve in Figure 3 provides the household with greater overall utility,  $u_1$ , than the higher bid curve,  $u_0$ .

Since the bid curve is simply an inverted indifference curve, the slope of the bid curve will be the same as the slope of the indifference curve but with the opposite sign i.e.  $U_{z_i}/U_x$ . And, of course, this ratio represents the left hand side of the condition for optimal residential location given in Equation (11).

So far our analysis has defined two closely related functions the indifference curve (Equation 13) and the bid curve (Equation 15). As yet, however, we have not determined

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<sup>9</sup> Note that Equation (14) does not enforce the budget constraint of Equation (10). In Equation (10)  $\theta$ , the amount the household could pay for a property, is replaced by  $P(\mathbf{z})$ , the amount that the household would have to pay in the market for a property.

<sup>10</sup> Though, clearly, the bid is constrained in that it cannot be greater than income,  $y$ .

how these functions are important in defining households' optimal residential choice. To do this, we must make use of the last of the first order conditions, that defining the budget constraint (Equation 10).

To plot the budget constraint we must rearrange Equation (10) to give;

$$x = y - P(z) \quad (10a)$$

In the left hand panel of Figure 4, the budget constraint has been added to the indifference diagram. The budget constraint describes all combinations of  $x$  and  $z_i$  that the household is able to buy given their income,  $y$ .<sup>11</sup> Bundles that are on the budget constraint or bundles that are to the left and below the budget constraint are affordable to the household. Those that are above and to the right of the budget constraint are too expensive for the household to purchase. In order to maximise their utility the household must choose amongst the affordable bundles of  $x$  and  $z_1$ . This amounts to choosing the bundle amongst affordable combinations of  $x$  and  $z_1$  that lies on the highest indifference curve. The optimal bundle  $(\hat{x}, \hat{z}_1)$  will be defined as the point of tangency between this highest indifference curve and the budget constraint. Notice that throughout this document we use a hat to represent a chosen bundle. Any other bundle in the affordable set will lie on an indifference curve that provides a lower level of utility.

The left hand panel of Figure 4 will be familiar to students of economics. However, the diagram differs from the usual consumer choice problem in that the budget constraint is not linear. For most goods marginal prices are constant, such that purchasing one more unit of a good will require a constant sacrifice in terms of ability to purchase other goods. Thus in the classic consumer choice problem, the budget constraint can be represented by a straight line. As we have already established, marginal prices may not be constant in hedonic analysis giving rise to the unfamiliar shape of the budget constraint.

The choice of an optimal bundle of housing attributes can just as easily be presented in terms of bid functions. Of course we have to transform the constraint to be expressed in the same terms as bids. Remember that the vertical axis of the bid function graph is measured in terms  $y - x$ ; that is money available to spend on housing attributes. Rearranging the income constraint, Equation (10), gives;

$$y - x = P(z) \quad (10b)$$

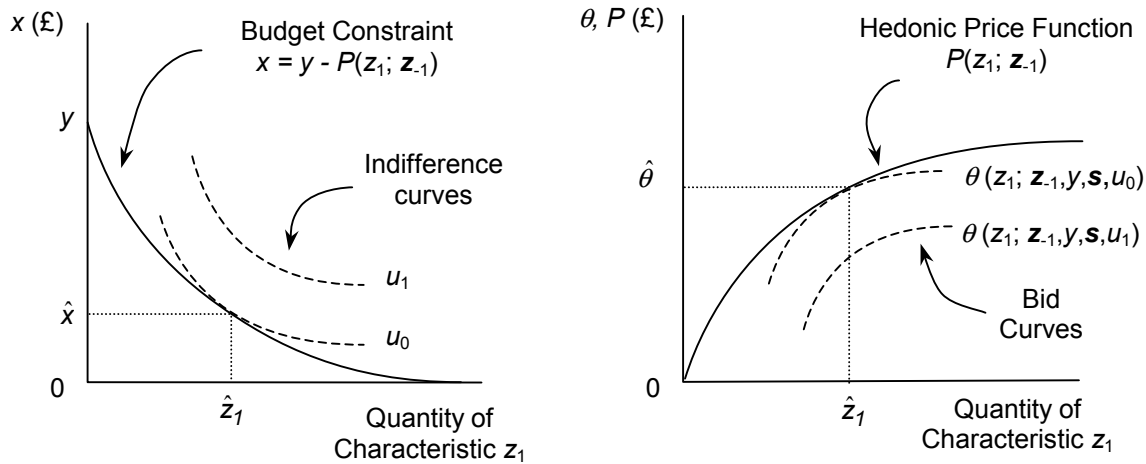
In other words, the relevant constraint is simply the hedonic price function. This is a very intuitive result. Bid functions reveal the amount that a household is willing to pay for

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<sup>11</sup> Again in order to illustrate the optimal conditions graphically we are forced to present a two-dimensional analysis. The figures presented here assume that all other characteristics of the property are unchanging in the analysis.

different levels of attribute  $z_1$ ; the hedonic price function gives the minimum price that they must pay in the market to purchase different levels of  $z_1$ .

**Figure 4: Choice of Optimal Residential Location using Indifference Curves and the Bid Function**



Intersections between bid curves and the hedonic price function indicate bundles of housing attributes at which the household's willingness to pay for a property with that bundle of attributes is equal to its market price. In maximising their utility, the household will choose the bundle of housing attributes that positions them on the bid curve providing the highest level of utility whilst still being compatible with reigning market prices. To be explicit, the household will maximise their utility by moving to the lowest bid curve that is just tangent with the hedonic price function.

The point of tangency between this lowest bid curve and the hedonic price function defines the bundle of housing attributes that fulfil the first order conditions for an optimal choice (Equations 7, 8 and 9).

Combining Equations (8) and (9) into the single expression in Equation (11), provides the first condition for an optimal choice. In particular this condition states that at an optimum, the slope of the bid function and the slope of the hedonic price function must be the same. That is, the household's willingness to pay to attain a property with one more unit of  $z_i$ ,  $U_{z_i}/U_x$ , must equal the market price of that extra unit,  $p_{z_i}$ . Thus a household's optimal choice of residential location will be one at which the value the household derives from the last unit of each housing attribute is exactly equal to the implicit price it had to pay for that unit. If this were not so then the household could increase their flow of utility by choosing an alternative property with different levels of attributes.

The second condition defining optimal choice is provided by the budget constraint, Equation (10). This states that the chosen bundle of attributes must be purchased at the

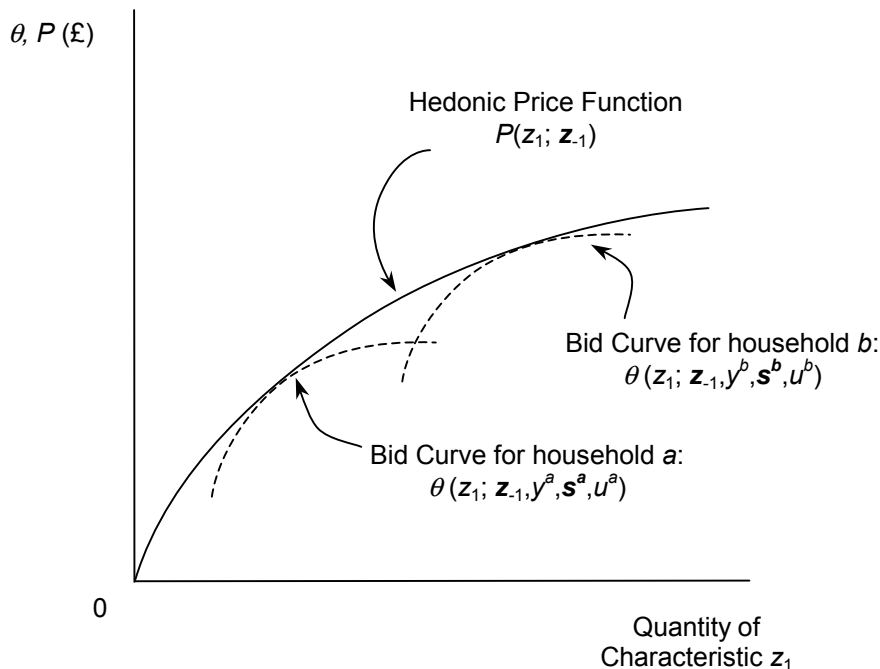
market price as defined by the hedonic price schedule. Combining this constraint with Equation (11) identifies the tangency point presented in Figure 4.

Of course, households do not have the same income nor do they have the same socioeconomic characteristics. Since both these arguments enter the bid function, we would expect bid curves to differ across households with different characteristics. Bid curves for two different households, denoted  $a$  and  $b$ , are illustrated in Figure 5. Notice that the conditioning arguments,  $y$  and  $s$ , are superscripted with this household indicator, showing that their values are specific to the particular household.

Again the optimal choice of property for each household will be defined by the tangency of one of their bid curves with the hedonic price function. Since the bid curves for the two households are different, the attributes of the property defined by this point of tangency will also differ. Notice that the utility level,  $u$ , that defines the optimising bid curve is also superscripted by a household indicator. Clearly, there is no reason to expect that the level of utility achieved by the two households would be the same.

If we were to add to Figure 5 the optimising bid curves for all the households in the market we would find that they were all tangential to the hedonic price function. Variation in household characteristics would mean that these points of tangency defined properties with different levels of the various housing attributes. In the terminology of economics, the hedonic price function forms an upper envelope to these optimising bid functions<sup>12</sup>.

**Figure 5: Choice of Property Attributes for Different Households**



<sup>12</sup> We can think of the hedonic price function almost like an envelope into which each of the optimising curves is fitted.

### *e. The Property Market: Landlord Choice*

So far we have examined the property market solely from the demand side; that is, in terms of consumers choosing between differentiated products. Though this decision is of greater interest for our present research objectives, it is worth taking some time to examine the supply side of the market; that is, to describe how landlords make their decisions concerning the type of properties to supply.

To simplify the analysis let us assume that each landlord rents out only one property.<sup>13</sup> In each period of time a household pays the landlord rent in order to live in this property. Of course this rent does not represent pure profit to the landlord. The landlord incurs costs in supplying this property for rental;

- First and foremost, through the initial purchase of the property.<sup>14</sup>
- Second, through maintaining the quality of the property by constant renovation and maintenance.
- Finally, through investments or disinvestments designed to change the attributes of the property subsequent to its purchase.

To incorporate these costs into our per period model, all discrete investments must be converted to equivalent per period costs. For example, the purchase price of the property can be expressed as an equivalent series of per period payments using Equation (2a).<sup>15</sup> In the same way, it is possible to express a discrete investment in the property, say the installation of double-glazing, as the discounted sum of a series of smaller equal-sized costs made over the expected lifetime of the investment.

The per period cost to the landlord of supplying a property with characteristics  $z$ , is given by the cost function<sup>16</sup>;

$$c(z; \hat{P}(\hat{z}), \bar{z}, r) \quad (16)$$

The cost of producing a property with characteristics  $z$  will differ across landlords for a number of reasons. The factors determining these differences are captured in the three conditioning arguments entering the cost function. These are;

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<sup>13</sup> The analysis remains relatively simple if we assume that the landlord rents out more than one property but that each property has the same characteristics. However, if the landlord rents out several properties with differing characteristics the model becomes considerably more complex whilst adding little to our understanding of the workings of the hedonic market.

<sup>14</sup> Or even the initial construction of the property though we ignore this possibility here.

<sup>15</sup> Indeed, in the UK, this does not represent a major abstraction from reality. It is typical for landlords to take out a mortgage in order to purchase a property. The purchase price of the property (plus interest on money borrowed) is repaid, therefore, in a series of monthly instalments.

<sup>16</sup> This cost function is the result of a minimisation problem in which the landlord attempts to find the cheapest cost means by which to produce a property with characteristics  $z$ .



- $\hat{P}(\hat{z})$ , which defines the *price paid for the property when first purchased*. Consider two landlords owning two originally identical properties. In terms of the notation, the original vector of housing attributes,  $\hat{z}$ , is identical for both landlords. Now, imagine that these two landlords purchased their properties at different times; the first during a recession that depressed the housing market and resulted in relatively low implicit prices for housing attributes, the second during a property market boom when implicit attribute prices were relatively high. Differences in the market price paid for the two properties are captured in differences in  $\hat{P}(\cdot)$ , the hedonic price function faced by the landlord at the time of purchase. Clearly, the cost of supplying the property is lower for the first landlord who bought when house prices were depressed than for the second landlord who bought when house prices were high.

Since this cost (expressed in equivalent per period terms) is constant for each landlord and independent of changes in the property market or in the characteristics of the property, we suppress this argument in the cost function henceforth.

- the vector  $\bar{z}$ , which defines the levels of attributes that, following the initial property purchase, are *provided costlessly to the landlord*.
  - For *structural attributes* this vector is likely to be identical to  $\hat{z}$ , the vector of housing attributes purchased by the landlord. For example, having purchased a two-bedroom house, the landlord does not have to pay further in order to maintain this number of bedrooms in the property.
  - For *locational, neighbourhood and environmental attributes*, the levels of  $\bar{z}$  will tend to be determined by factors that are beyond the control of the landlord. In economics we would describe these as *exogenous* factors. For instance,  $\bar{z}$  would include a measure of the baseline level of crime in the area. This baseline level of crime is provided costlessly to the landlord in so much as it is exogenously determined by government spending on crime prevention.<sup>17</sup> Indeed, baseline levels of many non-structural attributes of the property will be provided costlessly to the landlord since they tend to exhibit the characteristics of public goods. Other examples include the property's proximity to recreational facilities, its access to public transport, levels of air pollution and levels of noise pollution.

For many non-structural attributes, therefore, the values in  $\bar{z}$  will not be determined solely by  $\hat{z}$  (the vector of housing attributes initially purchased by the landlord). Indeed, to a large extent, the values of  $\bar{z}$  for non-structural attributes of the property are determined by public policy. Policies that reduce crime, redirect traffic, combat air pollution or increase the quality of public transport will determine the values included in  $\bar{z}$  for certain attributes. As we shall discuss in the next chapter, the primary aim of hedonic analysis is to determine the benefits to households and landlords of public projects that change the levels of these exogenously determined housing attributes.

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<sup>17</sup> Of course, the landlord indirectly pays for public goods such as these through taxation but payments are not directly linked to the level of the attribute enjoyed at the property and payments cannot be unilaterally altered so as to influence the level of attribute enjoyed at the property.

It is relatively easy to see why the vector  $\bar{z}$  is important in determining how much it costs landlords to achieve a certain level of attribute provision at their properties.

- Consider a *structural attribute*, such as the cost to two different landlords of providing a property with two bathrooms. Imagine that the properties owned by these two landlords are identical, except that the second landlord's property was purchased with a ground floor extension fitted with a second bathroom.<sup>18</sup> Clearly, it costs this landlord nothing to provide a two-bathroom property. On the other hand, the first landlord must invest in the building and fitting of the second bathroom. Clearly, the landlord's costs for providing a property with a certain level of structural attributes will be determined in part by the property's original level of structural attributes.
- Alternatively let us examine an *attribute with a public good element*. Take the peace and quiet attribute of a property as an example. Imagine two landlords, one whose property is in a quiet cul-de-sac and another whose property abuts a busy main road. For the sake of argument, we shall assume that their properties are identical in every other way. Now consider how much it would cost these two different landlords to achieve a certain level of peace and quiet. The landlord whose property is in a quiet road will have to spend little to attain a relatively high level of peace and quiet in the property. Possibly the most that would be needed would be to plant a few trees in the front garden to act as a barrier to traffic noise. Conversely, a landlord whose property is on a busy main road would have to invest relatively heavily in order to attain the same level of the peace and quiet attribute. Perhaps this landlord would need to install sound-proofing double-glazing. Clearly, the cost of attaining a certain level of peace and quiet will differ for the landlords of these otherwise identical properties depending on the exogenously determined level of traffic noise.
- the vector  $\mathbf{r}$ , captures *other parameters* important in determining the landlord's costs. For example,  $\mathbf{r}$  will include the characteristics of the landlord and the market price of investments. To illustrate, a landlord that is a capable plumber may be able to improve the quality of a property by installing an electric shower unit. The costs may be lower to this landlord than to another who has to seek professional help to achieve the same improvement.

The cost function, therefore, determines the per period cost of supply of a property with characteristics  $\mathbf{z}$ , given the purchase price of the property  $\hat{P}(\hat{\mathbf{z}})$ , the levels of exogenously determined property attributes,  $\bar{\mathbf{z}}$ , and a number of other parameters including the characteristics of the landlord,  $\mathbf{r}$ .

Importantly, landlords have the ability to change the characteristics of their property. For example, a landlord may choose to increase the peace and quiet at the property. For the

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<sup>18</sup> Obviously, since the two properties are not identical, we would expect their purchase prices to differ. However, this cost difference will be determined by the (possibly different) hedonic price functions faced by the two landlords at the time of purchase. These costs are already captured in the cost function by the argument  $\hat{P}(\hat{\mathbf{z}})$ .

sake of argument let us assume that the least cost method of achieving this increase is to install double-glazing. In this case the difference in the cost function evaluated at the original level of peace and quiet and the cost function evaluated at the increased level of peace and quiet would be the cost of the double-glazing.<sup>19</sup>

Given the per period cost defined by the cost function, the profit that a landlord derives from renting a property with characteristics  $\mathbf{z}$ , will be determined by the rental price the landlord can charge for such a property in the market. Hence;

$$\pi(\mathbf{z}; \bar{\mathbf{z}}, \mathbf{r}) = P(\mathbf{z}) - c(\mathbf{z}; \bar{\mathbf{z}}, \mathbf{r}) \quad (17)$$

Where  $\pi(\cdot)$  is the profit function defining the landlord's profit per period.

To make our analysis compatible with that for the demand side of the market, let us define a function that joins all combinations of  $\mathbf{z}$  and  $P(\mathbf{z})$  that return the same profit for the landlord. To do this, set  $\pi(\mathbf{z}; \bar{\mathbf{z}}, \mathbf{r})$  equal to the constant  $\pi$  in Equation (17). Then solve for the market prices that would be required in order to realise the profit  $\pi$  for different levels of  $\mathbf{z}$ . Mathematically, this amounts to;

$$\phi(\mathbf{z}; \bar{\mathbf{z}}, \mathbf{r}, \pi) = \pi + c(\mathbf{z}; \bar{\mathbf{z}}, \mathbf{r}) \quad (18)$$

This function is what Rosen terms the *offer function*. Simply put, the offer function describes the rent the landlord would need to receive in order to achieve a profit of  $\pi$  if he were to provide his property with a level of characteristics given by the vector  $\mathbf{z}$ .

As with their counterpart, bid curves, the offer function can be illustrated as *offer curves*. Each offer curve combines rental prices and levels of attribute provision that result in the same level of profit. The left hand panel of Figure 6 plots one landlord's offer curves for attribute  $z_1$ . The upper offer curve represents combinations of attributes and prices that would return a profit of  $\pi_2$ , the middle curve combinations giving a profit of  $\pi_1$  and the lower curve a profit of  $\pi_0$ .

As we would expect, higher offer curves define higher levels of profit for the landlord. Take for example one particular level of provision of  $z_1$ , let us say  $z_1^*$ . At this level of provision, the offer curves illustrated in Figure 6 evaluate to the three different offers  $\phi_0$ ,  $\phi_1$  and  $\phi_2$ . Since  $\mathbf{z}$ ,  $\bar{\mathbf{z}}$ , and  $\mathbf{r}$  are identical for each of these evaluations, the costs of provision must also be identical. All that changes between these offers is the level of

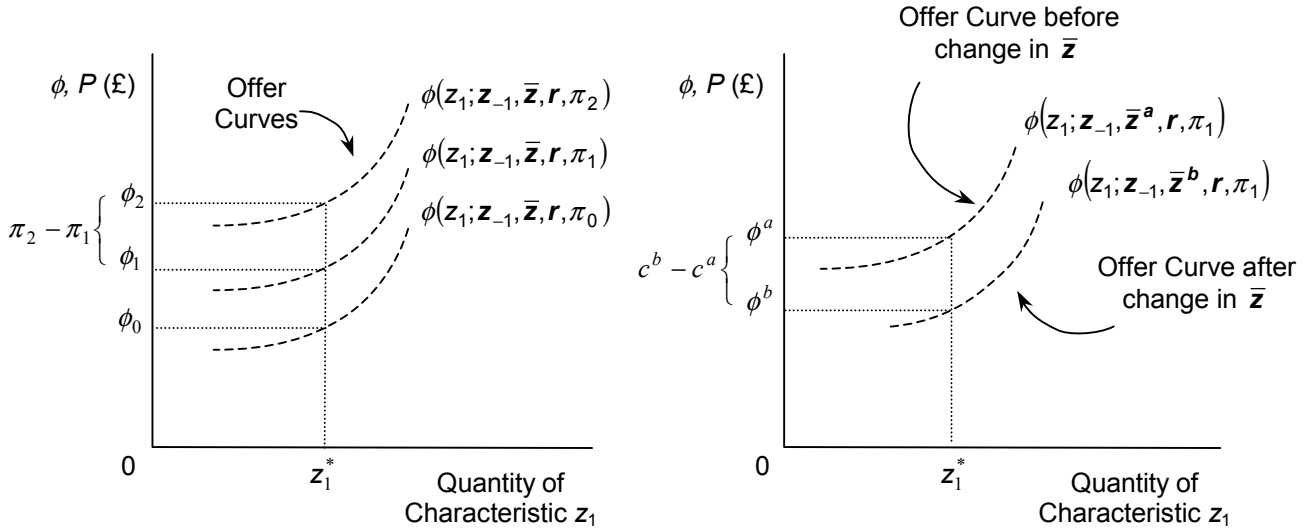
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<sup>19</sup> An investment such as the installation of double-glazing would require a one-off payment. Of course, it is possible to express this one-off payment as the discounted sum of a series of smaller equal-sized costs made over the expected lifetime of the investment. The increase defined by the cost function would be equal to this extra per period cost.

profit accruing to the landlord. Thus the vertical distance between two offer curves (in which the arguments entering the cost function remain unchanged) measures the difference in profit associated with the two curves. Of course, this should be evident from Equation (18).

In the left hand panel of Figure 6, therefore, the vertical distance between the middle and upper offer curves is the difference in profits associated with the two curves i.e.  $\pi_2 - \pi_1$ .

**Figure 6: The landlord’s offer curves**



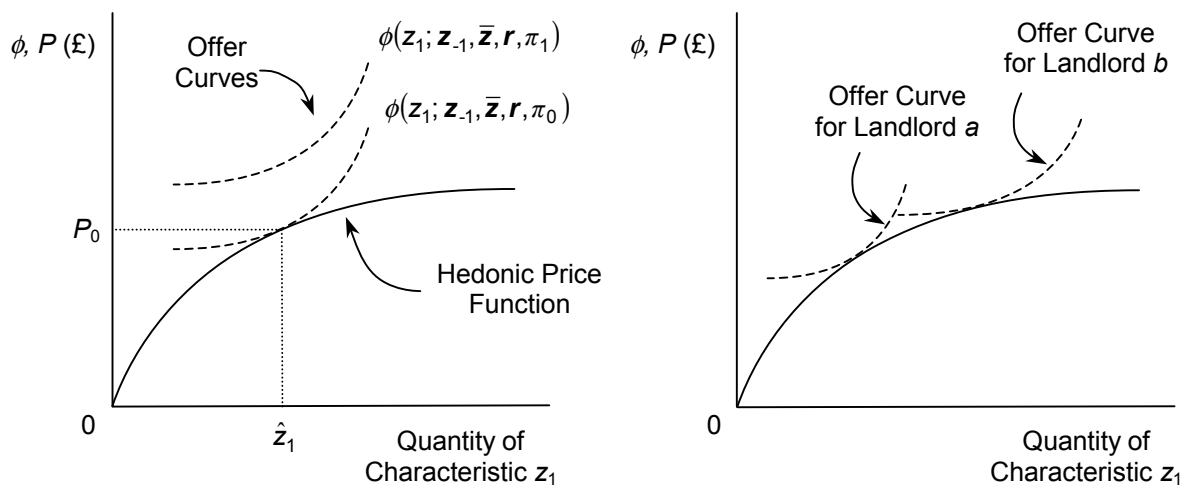
The right hand panel of Figure 6 depicts a comparable analysis, but this time we compare offer curves that differ in the level of the exogenously determined levels of  $\bar{z}$ . For example, we might imagine that the only difference between the upper and lower offer curves is the baseline level of crime in the area around the property. The upper curve is an offer curve for the landlord when faced by a low level of crime the, lower offer curve would represent the landlord’s situation faced by a higher level of crime. Notice that the two offer curves are associated with the same level of profit to the landlord,  $\pi_1$ . At a particular level of provision of provision of  $z_1$ , again let us focus on  $z_1^*$ , these two curves evaluate to two different offers,  $\phi^a$  and  $\phi^b$ . Clearly, the change in crime has influenced the costs of the landlord in providing a property with attribute levels  $z_1^*, z_{-1}$ . For example, the lower the level of crime, the lower the level of vandalism, the less the landlord would have to spend in maintaining and repairing the property. Since the two offer curves represent the same profit to the landlord, the vertical distance between them must indicate this cost saving, i.e.  $c^b - c^a$ . Notice also that this cost saving does not necessarily have to result from costs incurred in the provision of attribute  $z_1$ .

In maximising profit, the landlord seeks to provide the bundle of housing attributes that positions them on the offer curve providing the highest level of profit whilst still being compatible with reigning market prices. Similar to the choices made by households, this entails a tangency condition. In the left hand panel of Figure 7, the highest offer curve

compatible with the hedonic price function is that returning a profit of  $\pi_0$ . The best this landlord can achieve, therefore, is to alter the level of attribute  $z_1$  to  $\hat{z}_1$ , and charge a rent of  $P_0$ .

Again offer curves differ across landlords due to differences in purchase prices (though, for simplicity, this argument is suppressed in the analysis), the vector of parameters  $r$  and the exogenously determined levels of attributes  $\bar{z}$ . As a consequence, different landlords will choose to supply properties with different bundles of attributes. This is illustrated in the right hand panel of Figure 7. Indeed, the hedonic price function forms the lower envelope to the set of all landlords' optimising offer curves.

**Figure 7: Landlord's Optimising Choices of Housing Attributes to Supply**



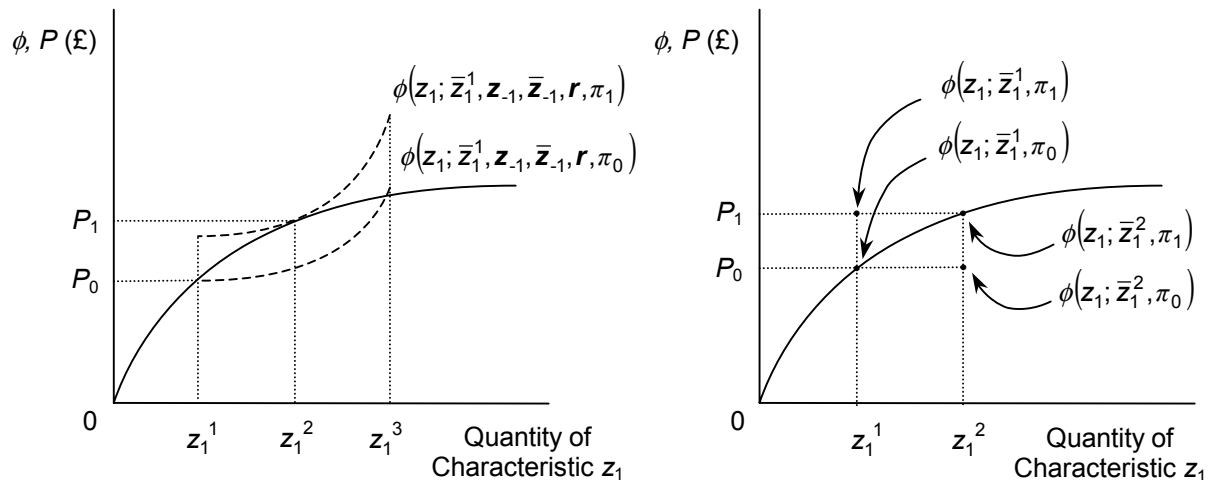
Offer curves differ slightly from their counterpart, bid curves, in so much as they will frequently be defined over quite a small range of attribute space. That is to say, for any one landlord the ability to change the attributes of the property may be relatively limited.

Let us return to our example of the peace and quiet attribute of a property. To a large extent, this is fixed for the property by exogenous factors, most notably proximity to noisy roads. The left hand panel of Figure 8 shows offer curves for a landlord whose property is exposed to an exogenously determined level of peace and quiet,  $z_1^1$ . As such, this quantity enters the offer function where it is labelled  $\bar{z}_1^1$ . At this level of the attribute the landlord's property would command a rent of  $P_0$  that would reward the landlord with a profit of  $\pi_0$ .

However, this is not the most profit that the landlord could earn on this property. As mentioned previously, the landlord could increase the level of peace and quiet at the property by, for example, planting trees in the front garden to act as a barrier to traffic noise and/or fitting sound-proofing double-glazing. The best the landlord could do is to increase the peace and quiet attribute of the property to  $z_1^2$ . Here he would be able to charge a higher rent,  $P_1$ , and would enjoy a profit of  $\pi_1$ .

In the example shown in the left hand panel of Figure 8, no amount of investment will increase the peace and quiet at this property beyond  $z_1^3$ , such that the offer function is not defined for values of  $z_1$  in excess of this quantity. As it happens, these restrictions will not concern this landlord as he maximises his profit by providing a property with  $z_1^2$  of the attribute.

**Figure 8: Landlords' Optimising Choice when the level of provision of an attribute is constrained**



However, the possibility arises that landlords may face what are termed corner solutions. That is, the profit maximising level of provision of a housing attribute may be at one of the constraints of the offer curve. To illustrate with our current example, the offer curve is constrained by the exogenously determined level of ambient noise pollution, which places a lower limit of  $z_1^1$  on the peace and quiet of the property, and by the limitations of noise avoidance technology, which places an upper limit of  $z_1^3$  on the peace and quiet of the property. If either of these limits were the profit maximising solution for the landlord then the simple tangential condition shown in Figure 7 would not hold.

In the extreme, landlords may have no control over the level of an attribute. In the terminology of the last paragraph the lower and upper limits for an attribute are one in the same. Take for example the proximity of the property to local amenities such as a shopping centre or school. Since the property is fixed in space there is no way in which the landlord can influence the time it would take a household living in that property to access these facilities.<sup>20</sup> In this case, the bid curves will shrink to a point above the exogenously determined level of the attribute.

<sup>20</sup> Once again, this is an example of an attribute whose value is exogenously determined and would enter the cost function and hence offer function in the vector  $\bar{z}$ .

Such a situation is illustrated in the right hand panel of Figure 8. Again to illustrate in one dimension we assume that the levels of all other property attributes do not change and for ease of exposition we suppress the vectors  $z_{-1}$ ,  $\bar{z}_{-1}$  and  $r$ , from the offer function. Consider first the situation where the level of attribute  $z_1$  is exogenously constrained to a level  $z_1^1$ . The best this landlord can do is to charge a rent of  $P_0$ , which returns a profit of  $\pi_0$ . Charging anything lower would necessitate missing out on possible profits, charging anything greater would make it impossible to rent out the property.

The only way in which this landlord could increase profits would be if there were an exogenous change in the level of  $z_1$  enjoyed at the property. For example, if attribute  $z_1$  represents accessibility to the town centre, then the building of an urban tram link that passed near the property would increase the accessibility of the property and consequently increase the level of attribute  $z_1$ . In Figure 8, this is represented by an increase from  $z_1^1$  to  $z_1^2$ .

If we assume that this change doesn't influence the cost of providing other property attributes then the landlord could continue to charge  $P_0$  and earn a profit of  $\pi_0$ . Of course, as illustrated in the figure, the landlord could make more profit than this by increasing the rent on the property to  $P^1$ . Charging this rent the landlord's profits increase to  $\pi_1$ .

We have dwelt on corner solutions such as those illustrated in the right hand panel of Figure 8 because such solutions typify environmental attributes and it is these attributes that are our central concern.<sup>21</sup> However, it is fair to assume that in the short run the levels of all attributes are constrained in a similar manner. For example, given the hedonic price function, it may increase the profitability of a particular property if it were to possess an extra bedroom. Of course, such changes do not happen overnight. The landlord might have to employ an architect to design an extension to the property, apply for planning permission, employ builders and finally have the proposed extension constructed, decorated and furnished. We shall return to such considerations in the next chapter.

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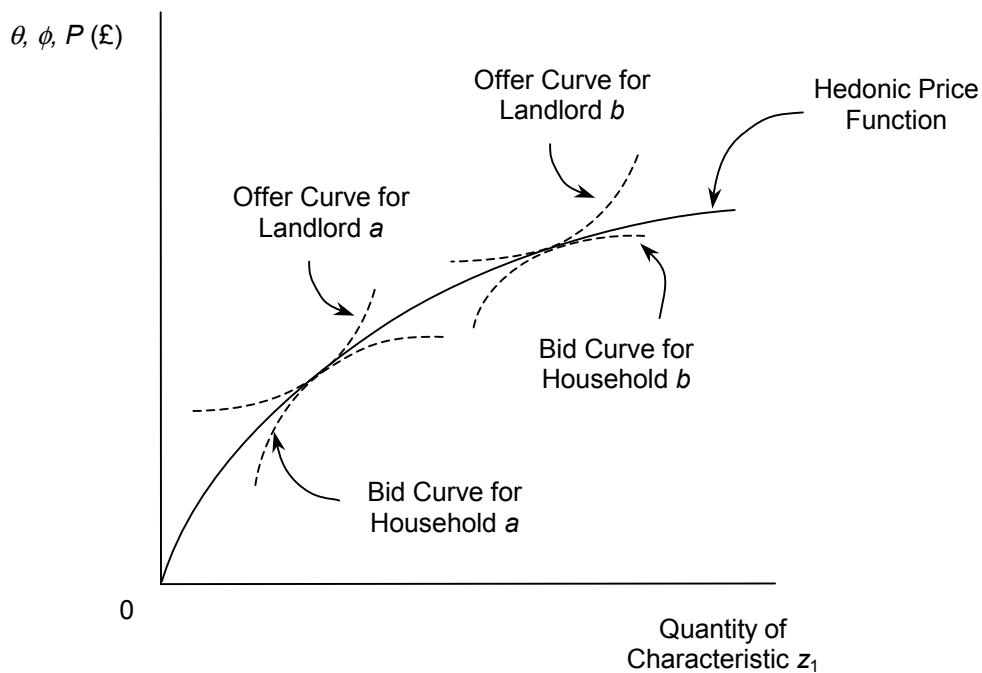
<sup>21</sup> Since many environmental qualities have the properties of public goods, their level of provision tends, to a greater extent, to be determined exogenously. Further, landlords frequently have limited scope for adjusting the levels of these attributes through investment in private goods for the property.

### ***f. The Property Market: Equilibrium***

So far we have examined the choices of consumers (households) and suppliers (landlords) in the property market independently. Figure 9 presents both sets of decisions combined in the same diagram. Households define their optimal residential location by choosing a property that boasts the set of attributes that coincide with the tangent of their lowest bid curve with the hedonic price function. The household cannot increase their utility by bidding for a property with different characteristics. Simultaneously, landlords maximise their profits by choosing to supply a property with the set of attributes that allows them to move to their highest offer curve that is still compatible with market prices. Supplying an alternative set of attributes would result in the landlord receiving offers for the property that resulted in lower levels of profits.

As depicted in Figure 9, the bid curves of households and the offer curves of landlords will “kiss” along the hedonic price function. At each coincidence of bid and offer curves, a landlord and household are paired; the landlord can do no better than to accept the household’s offer who in turn can do no better than to rent the property from that landlord.

**Figure 9: Choice of Property Attributes for Different Households**



The situation we have described is one of market equilibrium. At the reigning market prices revealed by the hedonic price schedule, demand for properties is equal to the supply of properties and the market clears. Hitherto our analysis has been at the level of individual households and landlords. At this disaggregate level we have assumed that each individual economic agent, being only a small player in the entire market, takes the equilibrium hedonic price schedule as given. To understand how the equilibrium is reached in the first place, we need to look at aggregate demand and supply.



Let us denote by  $Q^d(z)$  the aggregate market demand for properties with characteristics  $z$ . Similarly let  $Q^s(z)$  be the aggregate market supply for properties with characteristics  $z$ .<sup>22</sup> The market will be in equilibrium when the hedonic price function,  $P(z)$ , is such that  $Q^d(z) = Q^s(z)$  for all  $z$ . At this schedule of prices, individual landlords and households will act as has been described above.

Suppose that the quantities demanded and supplied for a particular combination of housing attributes, say  $z^*$ , do not match at the prevailing market prices. In such a case, we would expect market forces to act so as to change the hedonic price schedule and bring the market back into equilibrium. In the hedonic property market, however, this will not be so simple as to only involve change in the price of properties with attributes  $z^*$ . Instead, a price change for properties with this particular bundle of attributes will induce changes everywhere in the hedonic price schedule.

For example, imagine that excess demand for attributes  $z^*$  caused the price of properties with this bundle to increase. Households, who at the old hedonic price schedule would have maximised their utility by choosing a property with these attributes, will substitute away from that particular bundle thereby increasing demand for properties with different characteristics. Similarly, landlords noting the extra profits to be made by supplying properties with attributes  $z^*$ , will modify their properties to take advantage of the higher market prices. This will induce reductions in the supply of properties with alternative bundles of attributes. The process of substitution and relocation ripples through the entire market until a new hedonic price schedule is established that reconciles aggregate demand with aggregate supply for all  $z$ .

A number of researchers (e.g. Rosen, 1974; Epple, 1987) have attempted to analytically model equilibrium in hedonic markets. To do this it is necessary to make specific assumptions concerning the various behavioural functions that determine household and landlord behaviour. Specifically, researchers must assert a particular functional form for the utility function of households,  $u(\cdot)$ , and the cost function of landlords,  $c(\cdot)$ . Further they have to make assumptions concerning the distribution in the population of household characteristics,  $s$ , and landlord characteristics,  $r$ . Given specific forms for each of these different functions it should be possible to solve for an expression that gives the equilibrium hedonic price function. This expression will be a function of the arguments in the underlying functions. Hence, using such models it is possible to investigate how changes in the underlying arguments influence the hedonic price schedule.

Unfortunately, the complexity of the hedonic market is such that one must make very restrictive assumptions concerning the various functions in order to end up with an expression for the hedonic price schedule that is reasonably tractable. In general, therefore, research has concentrated on empirical analyses of hedonic markets.

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<sup>22</sup> If the attributes of properties cannot be influenced by landlords (as is probably true for many environmental attributes and for most property attributes in the short run) then the supply of properties with certain characteristics will be determined by the state of the current housing stock in the urban area. In this case aggregate supply will be perfectly inelastic and the equilibrium hedonic price schedule will be determined solely by the level of aggregate demand for properties with different characteristics.

In many ways, the problem for empirical analysis is the reverse of analytical research. Rather than assuming the functional forms of the underlying behavioural functions and working through the problem to solve for the hedonic price schedule, researchers estimate the hedonic price schedule from real world market data and work back through the problem to discover the form of the underlying behavioural functions. We shall return to how this might be achieved in Chapter 3.

