Nonparametric Eigenvalue-Regularized Precision or Covariance Matrix Estimator

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Abstract

Recently there are numerous works on the estimation of large covariance or precision matrix. The high dimensional nature of data means that the sample covariance matrix can be ill-conditioned. Without assuming a particular structure, much efforts have been devoted to regularizing the eigenvalues of the sample covariance matrix. We introduce nonparametric regularization of these eigenvalues through subsampling of the data. The subsampling idea for covariance matrix estimation is originally introduced in [1]. We improve on their covariance estimator, and for the first time provides vigorous proof that our version enjoys asymptotic optimal nonlinear shrinkage of eigenvalues with respect to the Frobenius error norm. Coincidentally, this nonlinear shrinkage is asymptotically the same as that introduced in [2]. One advantage of our estimator is its computational speed when the dimension p is not extremely large. Our estimator also allows p to be larger than the sample size n, and is always positive semi-definite. We prove that with respect to the Stein's loss function, the inverse of our estimator is the optimal precision matrix estimator. We also showed that all the aforementioned optimality holds for data with a factor structure as well, which can be useful in portfolio allocation. Our method avoids the need to estimate the unknown factors and factor loadings matrix first, and directly gives the covariance or precision matrix estimator. We compare the performance of our estimators with other methods through extensive simulations and a real data analysis.

Key words and phrases: High dimensional covariance matrix; Precision matrix; Regularized eigenvalues; Stieltjes transform; Subsampling.
Short title: Eigenvalue-Regularized Covariance Estimator

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