CORE VISCOITY

Definition of viscosity

The transport properties of fluid materials (either solid or liquid) are
determined by their viscosity, which defines the resistance of the mate-
rial to fluid flow (i.e., how "rarry" it is). Viscosity, $\eta$, describes the
time dependence of material motion through the ratio of applied shear
stress, $\sigma$, to strain rate, $\dot{\varepsilon}$ via

$$\eta = \frac{\sigma}{\dot{\varepsilon}}$$

(Eq. 1)

The Earth’s liquid outer core will have a relatively low viscosity com-
pared to that of the solid inner core, which deforms on a much longer
timescale. In geophysics, the quantity $\eta$ is referred to as the dynamic
viscosity, which, when normalized by the density, $\rho$, is termed the
kinematic viscosity, $\nu$:

$$\nu = \frac{\eta}{\rho}$$

(Eq. 2)

Viscosity is a quantity dependent on the properties of the fluid at a
molecular level. Therefore, both the viscosities defined above may also
be referred to as “molecular viscosity.” In the outer core, fluid motion
occurs over a range of length scales from that at a molecular level to
large-scale motion with characteristic distances comparable with the
outer core radius. Such large-scale motion does not exist in isolation,
but has embedded within it turbulence over a range of length scales,
which serve to drain the largest scale flow of its energy. Consequently,
this energy dissipation causes an increase in the viscosity of the large-
scale fluid, resulting in an “effective viscosity” or “turbulent viscosity”
of a much larger magnitude than the molecular viscosity defined
carlier. This effective viscosity has been observed experimentally in
turbulent fluids yet it is very difficult to quantify and impossible to
derive rigorously; however, estimates for effective viscosities are often
used when modeling the outer core. Viscosity is a very important para-
meter in geophysics since the viscosity of materials in the Earth’s core
are a contributory factor in determining overall properties of the core
itself, such as core convection (see Core properties and Core convec-
tion); indeed, the fundamental equations governing the dynamics of
the outer core and the generation and sustention of the magnetic field
are dependent, in part, on the viscosity of the outer core fluid.

Quantifying outer core viscosity

There have been many estimates made for outer core viscosity derived
from geodetic, seismological, geomagnetic, experimental, and theoreti-
cal studies. However, the values so obtained span 14 orders of magnitude
(see Secco, 1995).

Geodetic observations (e.g., free oscillations, the Chandler wobble,
length of day variations, rotation of the Earth, tidal measurements, grav-
itimetry) lead to viscosity estimates ranging from $10^{-6}$ mPa s (observa-
tions of the Chandler wobble, Verhoogen, 1974) to $10^{15}$ mPa s (analysis of
free oscillation data, Sato and Espinosa, 1967). Theoretical geodetic stu-
dies (e.g., viscous coupling of the core and mantle, theory of rotating
fluids, inner core oscillation (q.v.), core nutation) lead to viscosity esti-
mates ranging from $10^{-11}$ mPa s (evaluation of decay time of inner core
oscillation (q.v.), Won and Kuo, 1973) to $10^{-8}$ mPa s (secular deceller-
ation of the core by viscous coupling, Bondi and Lyttleton, 1948). Gener-
ally, much higher values for viscosity ($10^{10}$ mPa s to $10^{14}$ mPa s) are
obtained from seismological observations of the attenuation of P- and
S-waves through the core (e.g., Sato and Espinosa, 1967; Jeffreys, 1959),
and from geomagnetic data (e.g., $10^{10}$ mPa s, Officer, 1986).

The viscosities of core-forming materials may also be determined
experimentally in the laboratory and theoretically through computer
simulation. Empirically, viscosity follows an Arrhenius relation of
the form (see Poirier, 2002):

$$\eta \propto \exp \left( \frac{Q_v}{R T} \right)$$

(Eq. 3)

where $Q_v$ is the activation energy. Poirier (1998) analyzed data for a
number of liquid metals and found that there is also an empirical rela-
tion between $Q_v$ and the melting temperature:

$$Q_v \cong 2.6 R T_m$$

(Eq. 4)

This very important result implies that the viscosity of liquid metals
remains constant (i.e., independent of pressure) along the melting
curve and therefore equal to that at the melting point at ambient pres-
sure, which is generally of the order of a few mPa s. Furthermore,
Poirier went on to state that the viscosity of liquid iron in the outer
core would, therefore, be equal to that at ambient pressure ($\sim 6$ mPa s).

On a microscopic level, an approximation for the viscosity of liquid
metals is given by the Stokes-Einstein equation, which provides a
relationship between diffusion and viscosity of the form:

$$D \eta = \frac{k_B T}{2 \pi a}$$

(Eq. 5)

where $a$ is an atomic diameter, $T$ is the temperature, $k_B$ is the Boltz-
mann constant, and $D$ is the diffusion coefficient.

Theoretical values for diffusion coefficients have been obtained
from ab initio molecular dynamics simulations on liquid iron at core
conditions (de Wijs et al., 1998), leading to a predicted viscosity of
$\sim 12-15$ mPa s using the Stokes-Einstein relation above. However,
although the Stokes-Einstein equation has proved successful in estab-
lishing a link between viscosity and diffusion for a number of mono-
tropic liquids, it is not necessarily the case that it should be effective
for alloys or at high pressures and temperatures. To address the validity of the Stokes–Einstein relation and to assess the effect of imperfections on viscosity coefficients, a number of experimental and theoretical studies have been performed on the Fe–FeS system. High-pressure tracer diffusion experiments (Dobson et al., 2001) have been carried out on liquid Fe–FeS alloys at 5 GPa, resulting in high diffusivities (10⁻⁶ cm² s⁻¹) in excellent agreement with ab initio molecular dynamics calculations performed at the same conditions (Vočadlo et al., 2000). When incorporated into equations 5, these diffusivities lead to values for viscosity of a few mPa s.

Direct viscosity measurements (Dobson et al., 2000) of Fe–FeS alloys by means of the falling-sphere technique have been made at similar pressures and temperatures to those used in the diffusion experiments above; these resulted in values for viscosities in excellent agreement with those derived experimentally using the Stokes–Einstein relation. Furthermore these results are in excellent agreement with ab initio molecular dynamics calculations of viscosity based on treatment of the stress tensor derived from the simulations (Vočadlo et al., 2000). All of these results thus provide both experimental and theoretical verification of the Stokes–Einstein relation (equation 5).

The results from these studies further show not only that the viscosity of liquid iron at core pressures is approximately equal to that at low pressures but that the viscosity of low pressure, experimentally measured iron is a decreasing function of pressure, approaching the values of liquid iron at core pressures. However, this is likely to be a lower limit as the value may increase at higher pressures. Seismological and geodetic observations have been used to constrain the viscosity of the Earth's outer core. The results of these studies suggest that this quantity is the same at all core pressures, with the viscosity of the outer core being a few mPa s, while the viscosity of the inner core is likely to be even lower, on the order of mPa s (compare to that of water on the Earth's surface).

In general, viscosities derived from Earth observations are high (10⁻¹⁵–10⁻¹⁶ mPa s) and those based on laboratory experiments and theoretical considerations are much lower (~1–10 mPa s). The disparate values for viscosity arise for two main reasons: the type of viscosity being measured (molecular or effective) and the large uncertainties associated with the interpretation of Earth observation data. It is also possible that viscosity measurements made from the direct observations of the Earth are not actually measuring viscosity at all, but some other effects that are being attributed to viscosity; this would certainly be the case if the “true” viscosity were of the order of mPa s, since such a small viscosity is unlikely to be detectable. A low viscosity leads to viscous forces that are essentially negligible compared to the Coriolis force, supporting the view of an outer core in a state of small-circulation turbulent convection rather than a more coherent pattern of convection on a much larger scale comparable to the core radius. If this is the case, it is necessary to obtain accurate values for viscosity in order to quantify this turbulence in the outer core.

**Viscosity and the inner core**

The inner core is not perfectly elastic and has a finite viscosity with deformation occurring over long timescales. Placing numerical constraints on the viscosity of the inner core is fundamental to understanding important core processes such as differential inner core rotation (q.v.), inner core oscillation (q.v.), and inner core anisotropy (q.v.) (see Dumberry and Bloxham, 2002). In particular, Buffet (1997) modeled the viscous relaxation of the inner core by calculating the relaxation time for the inner core to adjust, as it rotates, to its equilibrium shape after small distortions due to perturbations in gravitational potential imposed by the overlying mantle. He suggested that the viscosity that has to be constrained to be either less than 10³ Pa s (if the whole inner core is involved in the relaxation) or greater than 10¹⁴ Pa s (if there is no relaxation of the inner core), although this latter case may lead to gravitational locking and hence no differential rotation.

Quantifying the viscosity of the phases present in the inner core at a microscopic level is a very difficult problem. At temperatures close to the melting point (as expected in the inner core) viscous flow is likely to be determined either by dislocation creep (Harper–Donn creep) or diffusion creep (Nabarro–Herring creep).

The overall viscosity of inner core material has diffusion-driven and dislocation-driven contributions:

\[ \eta = \left( \frac{1}{\eta_{\text{diff}}} + \frac{1}{\eta_{\text{dis}}} \right)^{-1} \]

(Eq. 6)

Diffusion-controlled viscosity, whereby the material strain is caused by the motion of lattice defects (e.g., vacancies) under applied stress, is given by

\[ \eta_{\text{diff}} = \frac{d^2 RT}{6\pi \mu d \nu} \]

(Eq. 7)

where \( d \) is the grain size, \( R \) is the gas constant, \( T \) is the temperature, \( \mu \) is a geometric constant, and \( \nu \) is the volume. The self-diffusion coefficient, \( D_{\text{sd}} \), is given by

\[ D_{\text{sd}} = D_0 \exp \left( -\frac{\Delta H}{RT} \right) \]

(Eq. 8)

where \( D_0 \) is a preexponential factor and \( \Delta H \) is the activation enthalpy for self-diffusion.

For simple materials, dislocation-controlled viscosity, whereby material strain is caused by the movement of linear defects along crystallographic planes, is given by

\[ \eta_{\text{dis}} = \frac{RT}{\mu D_{\text{sd}} \nu} \]

(Eq. 9)

where \( \rho \) is the dislocation density.

Both dislocation- and diffusion-controlled creep mechanisms are thermally activated and the thermally controlled parameter in both cases is the self-diffusion coefficient, \( D_{\text{sd}} \). A commonly used empirical relation for metals assumes that \( \Delta H \) is linearly proportional to the melting temperature, \( T_{\text{mol}} \), and hence that

\[ D_{\text{sd}} = D_0 \exp \left( -\frac{\varphi T_{\text{mol}}}{T} \right) \]

(Eq. 10)

where \( \varphi \) is a constant taking a values of ~18 for metals (Poirier, 2002).

Considering iron close to its melting point at core pressures (~5500 K), and using reasonable estimates for other quantities in equations 7, 8, and 10 (\( \varphi \approx 42; D_0\approx 10^{-3} \text{ m}^2 \text{s}^{-1} ; \nu \approx 5 \times 10^{-2} \text{ m}^2 \text{ mol}^{-1} \)), we obtain values for \( \eta_{\text{diff}} \) and \( \eta_{\text{dis}} \) of ~10²⁵ d² Pa s and ~6 × 10¹⁵ Pa s, respectively. Unfortunately, the strong dependence of the viscosity expressions on the completely unknown quantities of grain size and dislocation density means that it is extremely difficult to produce reliable final numerical values. Grain sizes in the inner core could be anything from 10⁻¹⁰ m, resulting in diffusion viscosities in the range 10¹⁵–10²⁴ Pa s; dislocation densities could be as low as 10⁴ m⁻² or nearer to the dislocation melting limit of 10²⁷ m⁻², resulting in dislocation driven viscosities of 10⁻⁶–10⁻³ Pa s. Thus, some of the relative contributions from dislocation controlled and diffusion controlled viscosity are as yet unknown.

Clearly, inner core viscosity is not a well-constrained property, with estimates varying over many orders of magnitude. Future microscopic simulations, combined with high-resolution seismic and geodetic data, should constrain this quantity further and thereby improve our understanding of inner core dynamics.

**Bibliography**


Cross-references
Core Convection
Core Properties, Physical
Core Properties, Theoretical Determination
Inner Core Anisotropy
Inner Core Oscillation
Inner Core Rotation