ELEMENTS OF PLATE TECTONICS

OUTLINE

ORIGIN AND HISTORY

BASIC CONCEPTS
  Rheology of the crust and upper mantle.
  Lithosphere and asthenosphere: lithospheric plates.
  Geometry of plate motion: Euler poles.
  Timing.

EVIDENCE FROM GEOPHYSICS
  – a tabulated summary.

EULER POLES
  Euler poles and motion on a sphere
  The geometry of mid-ocean spreading
  Euler poles in plate kinematics
  Reconstructions of past motion
  Worked example

Background reading: §2.1–2.4 and 3.3 in Fowler; §1.2 and 6.3.3 in Lowrie.
Plate tectonics is the theory that unified earlier ideas of *continental drift* and *seafloor spreading*. It can be subdivided into *plate kinematics*, which describes the motion of the plates, and *plate dynamics*, which deals with the forces acting on the plates. It is supported by a huge body of observation that eventually overcame resistance to its ideas. Plate tectonics allows plate motion to be calculated and verified in considerable detail back to the palaeozoic era. Yet what drives the plates is still in question.

The history of how plate tectonics became an accepted theory is one of the sagas of modern science. Chapter 1 of Kearey & Vine outlines its key stages. There are several more complete accounts such as the book by Hallam (1973, *A Revolution in the Earth Sciences*, Clarendon Press).
RHEOLOGY OF THE CRUST AND UPPER MANTLE

- Rheology is the science of deformation and flow. The deformation and flow properties of crustal and mantle rocks support the idea that the strength of the mantle drops sharply at about 80 km depth. Consequently slow movements above ~ 80 km are only weakly coupled to the deeper interior of the Earth.
- The lithosphere is the rigid outer shell of the Earth, comprising crust and outermost mantle, and the asthenosphere is the weak layer in the mantle starting at depths of about 80 km and extending to a few 100 km depth. The transition between the two is not sharp and, for the purposes of discussion, is often defined by an isotherm (e.g. the 1400°C isotherm) since it is essentially temperature that determines where mantle rocks lose their strength.
- Note that the boundary that is important in plate tectonics marks a change in strength. The boundaries between crust, mantle and core mark changes in composition.
- Time scale is important in rheology. Since the asthenosphere transmits shear waves, it retains some shear strength at the periods of oscillation of seismic waves but over times measured in years it behaves like a viscous fluid.
- Since the asthenosphere has no long-period shear strength, stress can be transmitted through the lithosphere over considerable distances.
BASIC IDEAS OF PLATE TECTONICS (2)

LITHOSPHERIC PLATES
• There is much evidence that the lithosphere is divided into more–or–less rigid plates. There are about 12 major plates, and many minor ones and platelets.
• The plates are more or less rigid and move relative to one another over geological time.
• Most plate boundaries are narrow. Geophysical measurements near plate boundaries (e.g. magnetic anomalies, seismicity, bathymetry) can determine the relative motion between neighbouring plates.

GEOMETRY OF PLATE MOTION
• The motion of lithospheric plates has to satisfy the geometrical constraints of rigid bodies moving on a sphere. These constraints impose an internal consistency on the directions and rates of motion at plate boundaries.
• Euler’s Theorem describes the motion of a rigid body on a sphere by means of a pole of rotation (Euler pole) and an angular velocity about the pole.

TIMING
• Dating allows rates of deformation and motion to be estimated.
• Radiometric dating can provide absolute timing but only to a few percent and at sampled locations. Sea–floor magnetic anomalies act as isochrons, providing continuous timing over huge areas which enables detailed mapping of plate motions.

Geometry and timing together make plate tectonics a quantitative and testable theory.
PRESSURE AND TEMPERATURE PROFILES IN THE MANTLE

The dashed lines mark the zone of partial melting.
The strength of the lithosphere depends on the composition of its rocks. The strength of a given type of rock increases with increasing confining pressure but decreases with increasing temperature. The diagrams below are schematic profiles of strength through oceanic and continental lithosphere. BDT denotes a brittle-to-ductile transition. While oceanic plates can generally be regarded as rigid, continental tectonics is more complicated and involves more detailed models.

Plate boundaries tend to fall into three types: constructive, destructive and conservative; these can alternatively be called divergent, convergent and transform fault plate boundaries.

The figure on the right shows schematic representations of the three types of plate boundary: F is a transform fault margin; R is a divergent plate margin; T is a convergent plate margin. The barbs are on the overthrust plate of the convergent plate margin, the arrow on the underthrust plate.

*Triple junctions* are places where three plates meet. The analysis of triple junctions is part of the Global Tectonics course.
CARTOON OF PLATE BOUNDARIES

- Oceanic crust
- Island arc
- Active volcano
- Ocean trench
- Benioff zone
- Earthquakes to a depth of 700 km
- Lithosphere
- Asthenosphere
- Transform fault
- Axial rift
- Ocean ridge
- Transform fault
- Lithosphere
- Asthenosphere
## CHARACTERISTICS OF PLATE BOUNDARIES

<table>
<thead>
<tr>
<th></th>
<th>divergent</th>
<th>transform</th>
<th>convergent boundaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>lithosphere</td>
<td>created</td>
<td>neither created nor destroyed</td>
<td>subduction or collision</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>sites of orogenesis and metamorphism</td>
</tr>
<tr>
<td>volcanoes</td>
<td>basaltic</td>
<td>none</td>
<td>subduction → andesitic</td>
</tr>
<tr>
<td>stresses</td>
<td>tensional</td>
<td>shear</td>
<td>complex, mainly compressional</td>
</tr>
<tr>
<td>earthquakes†</td>
<td>shallow</td>
<td>shallow</td>
<td>shallow, intermediate, deep</td>
</tr>
<tr>
<td>faulting</td>
<td>normal</td>
<td>strike–slip</td>
<td>varied, including thrusts</td>
</tr>
</tbody>
</table>

† Earthquake distribution:
Ocean ridge (ocean–ocean constructive margin): earthquakes in a narrow band
Rift valley (continent–continent constructive margin): earthquakes in a broad belt
Oceanic transform faults: earthquakes in a narrow band
Continental transform faults: earthquakes in a broad band
Ocean trench and island arc (ocean–ocean destructive margin): shallow to deep earthquakes in a dipping band
Ocean trench and cordillera (ocean–continent destructive margin): shallow, intermediate and possibly deep earthquakes in a dipping band
Collision zones (continental, continent–island arc, island arc collisions): shallow and possibly intermediate earthquakes in a broad zone
The table below lists information on plate tectonics provided by various sub-areas of Solid Earth Geophysics. This information is discussed in more detail during the course.

<table>
<thead>
<tr>
<th>Geophysical method</th>
<th>Contribution</th>
</tr>
</thead>
</table>
| Geodesy            | modern measurements of continental drift  
crustal movements, rates of uplift  
with gravity mass and moment of inertia of the Earth ⇒ mass distribution in the Earth  
geoid ⇒ mass distribution in the crust and mantle  
isostatic rebound ⇒ viscosity of the mantle |
| Gravity            | isostasy, mountain roots, subduction zones |
| Seismology         | internal structure of the Earth: crust, mantle, core.  
seismic velocities (→ elastic moduli and density)  
atteuation (absorptive zones, e.g. partial melting) |
| Seismicity         | delineation of plate boundaries and subducting plates (Benioff zones)  
earthquake mechanisms ⇒ lithospheric stresses |
| Magnetism          | conditions in the core and lower mantle  
reversals: sea-bed anomalies, magnetostratigraphy  
palaeomagnetism, apparent polar wander, plate motions in the past |
The following continues the table from the previous page. The topics in this list are not covered in this course.

<table>
<thead>
<tr>
<th>Geophysical method</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geochronology</td>
<td>timing</td>
</tr>
<tr>
<td></td>
<td>continental accretion, provenance of rocks</td>
</tr>
<tr>
<td>Electromagnetism</td>
<td>conductivity in the interior of the Earth</td>
</tr>
<tr>
<td>Heat flow</td>
<td>relates to sea-floor topography</td>
</tr>
<tr>
<td></td>
<td>differences in oceanic and continental crust</td>
</tr>
<tr>
<td></td>
<td>temperatures in the Earth’s interior</td>
</tr>
<tr>
<td></td>
<td>energy sources within the Earth</td>
</tr>
</tbody>
</table>
**EULER POLES**

*How can we describe the relative motion of lithospheric plates?*

Answer: by (1) an Euler pole and (2) *either* an angular velocity about the pole *or* a maximum spreading rate. Euler poles are explained below.

*How can we locate an Euler pole?*

This is illustrated by a worked example to locate the Euler pole for the Pacific and Antarctic plates from spreading rates and the orientation of transform faults on the Pacific–Antarctic ridge.

*How are Euler poles used?*

They are built into computer models of plate motion, such as *Time Machine Earth*, which allow the configuration of the continents to be projected back in time.
EULER POLES AND MOTION ON A SPHERE

The simplest kind of motion on a sphere is motion in a circle around an axis of rotation. The Euler pole corresponds to the point where the axis of rotation cuts the sphere; by convention, the positive pole is the one around which the rotation is anticlockwise when viewed from outside the sphere.

• Terminology: circles on a sphere are either
  1. *great circles*, which cut the sphere into 2 hemispheres (e.g. the equator, lines of longitude)
  2. *small circles*, which are all other circles (e.g. lines of latitude).

• The radius of a circle on a sphere is

\[ R \sin \Delta = R \cos \lambda \]

where \( R \) is the radius of the sphere, \( \lambda \) is the latitude of the circle with respect to the Euler pole and \( \Delta = 90^\circ - \lambda \) is the colatitude of the circle with respect to the Euler pole. \( \Delta \) is the geocentric angle (= angle at the centre of the Earth) between the circle’s axis of rotation and a point on the circle. For a great circle \( \Delta = 90^\circ, \lambda = 0^\circ \).

• The instantaneous motion of a body on a sphere can be specified by its Euler pole and its angular velocity \( \omega \) about that pole. The linear velocity of a moving body at colatitude \( \Delta \) relative to its Euler pole is

\[ v_\Delta = \omega R \sin \Delta \]

The linear velocity \( v_\Delta \) varies from 0 at the Euler pole to a maximum of \( \omega R \) at the Euler equator.

**NOTE:** \( \omega \) must be expressed in radians (e.g. radians per year) in these formulae.

*See the Appendix on measurement of angles if you are not familiar with radians or trigonometry.*
MOR = mid-ocean ridge; $\theta =$ strike of transform fault relative to North;
$Az =$ azimuth (bearing) relative to North of the Euler pole from the MOR site.
The MOR lies on a great circle through the site and the Euler pole. The transform fault is tangential to a small circle through the site and centred on the Euler axis through the centre of the Earth. ⇒ How are $Az$ and $\theta$ related?
SPREADING RATES

Relative to A, B lies on a small circle centred on D with radius $BD = R_E \sin \Delta$ where $R_E$ is the radius of the Earth. If A is an Euler pole, then the spreading rate per year at B is $\omega R_E \sin \Delta$ where $\omega =$ the angular velocity of spreading in radians/year.

⇒ What is the maximum spreading rate and where does it occur?

$\Delta =$ geocentric angle between A and B

great circle section through sites A and B

(C = centre of the Earth)

Cross section through the centre of the Earth showing the geocentric angle $\Delta$
EULER POLES IN PLATE KINEMATICS

Motion of plates on a sphere

• In practice plate motions are measured relative to other plates. The motion of one plate relative to another is specified by
  (a) the Euler pole of relative plate rotation, and
  (b) the relative angular velocity about that pole (or a maximum spreading rate).
• Plate boundaries perpendicular to small circles are either convergent (constructive) or divergent (destructive) depending
  on whether the relative motion is towards or away from the small circle.
⇒ The rate of divergence at a symmetrically spreading ridge varies as $v_90 \sin D$ where $v_90$ is the maximum spreading rate,
  i.e. the spreading rate at the Euler equator.
Note: in practice it is hard to measure rates of convergence at a trench.
• Plate boundaries parallel to small circles are conservative, i.e. transform faults.
⇒ Great circles perpendicular to pure transform faults pass through (in practice, close to) the Euler pole.
These properties are the key to the worked example on locating the present-day Euler pole of Europe and North America.
There are essentially three ways of reconstructing the past configurations of the continents:

1. by fitting continental margins (e.g. Bullard, Everett & Smith 1965);
2. by undoing seafloor spreading, i.e. reversing the tracks of continents revealed by the isochrons from seabed magnetic anomalies; see Fowler, Figure 3.15;
3. by inverting apparent polar wander (APW) paths, i.e. by finding the motion of the continents that best explains observed APW.

The third method is the least accurate but it is the only method for ages in excess of 160 Ma or so. The results are condensed to a series of Euler poles and associated angular velocities sufficient to fit the observations.

Mapping past motions
- Section 3.3 in Fowler (1990) summarises past plate motions of the Atlantic, Indian and Pacific Oceans and the continents back to Pangaea (late Carboniferous, 280 Ma).

References at the end of Fowler’s Chapter 3 (e.g. Briden, Hurley & Smith 1981, Owen 1983, Scotese et al. 1979 & 1988, Smith, Hurley & Briden 1981) include a number of papers showing world maps at various times.
- The computer program “Atlas” from Cambridge Paleomap Services simulates plate motion back in time and is widely used in research.
WORKED EXAMPLE ON LOCATING AN EULER POLE (1)

Estimation from the variation in spreading rate

The table below lists spreading rates $V$ measured from seafloor magnetic anomalies at five sites along the Pacific-Antarctic ridge from data in Chase (1978). The values are full spreading rates, not half rates. The table also lists the geocentric angles ($D_1$, $D_2$, $D_3$) in degrees to each site from three trial Euler poles (1, 2, 3) for the motion of the Pacific relative to Antarctica.

<table>
<thead>
<tr>
<th>site</th>
<th>latitude $^\circ$S</th>
<th>longitude $^\circ$W</th>
<th>$V$ km/Ma</th>
<th>$D_1$ $^\circ$</th>
<th>$D_2$ $^\circ$</th>
<th>$D_3$ $^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>35.6</td>
<td>110.5</td>
<td>111</td>
<td>76.4</td>
<td>78.3</td>
<td>74.4</td>
</tr>
<tr>
<td>b</td>
<td>44.5</td>
<td>112.0</td>
<td>102</td>
<td>67.5</td>
<td>69.3</td>
<td>65.4</td>
</tr>
<tr>
<td>c</td>
<td>51.2</td>
<td>118.5</td>
<td>97</td>
<td>59.9</td>
<td>61.7</td>
<td>58.1</td>
</tr>
<tr>
<td>d</td>
<td>59.0</td>
<td>151.0</td>
<td>78</td>
<td>45.4</td>
<td>47.0</td>
<td>44.7</td>
</tr>
<tr>
<td>e</td>
<td>64.0</td>
<td>169.0</td>
<td>63</td>
<td>36.3</td>
<td>37.7</td>
<td>36.0</td>
</tr>
</tbody>
</table>

The latitudes and longitudes of the three Euler poles are
1 : (66.2$^\circ$S, 96.5$^\circ$W); 2 : (64.3$^\circ$S, 96.0$^\circ$W); 3 : (69.2$^\circ$S, 90.3$^\circ$W).

Procedure:
(i) Calculate a table of values of the ratio $V/\sin D$ where $D$ is the angular distance.
(ii) For each pole, calculate the average value of $V/\sin D$.
   This average corresponds to the maximum spreading rate: can you see why?
(iii) For each pole, calculate the deviations from the average and decide which pole fits the observations best.

WORKED EXAMPLE ON LOCATING AN EULER POLE (2)

Estimation from the strike directions of transform faults

The following table gives the strike of transform faults observed at another five sites along the Pacific-Antarctic ridge together with the calculated azimuths, or bearings, ($A\alpha_1$, $A\alpha_2$, $A\alpha_3$) in degrees from each site to the three trial Euler poles.

<table>
<thead>
<tr>
<th>site</th>
<th>latitude °S</th>
<th>longitude °W</th>
<th>strike °</th>
<th>$A\alpha_1$ °</th>
<th>$A\alpha_2$ °</th>
<th>$A\alpha_3$ °</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>52.9</td>
<td>118.8</td>
<td>108.0</td>
<td>16.0</td>
<td>16.7</td>
<td>12.0</td>
</tr>
<tr>
<td>w</td>
<td>55.0</td>
<td>126.5</td>
<td>111.0</td>
<td>19.8</td>
<td>20.7</td>
<td>15.5</td>
</tr>
<tr>
<td>x</td>
<td>64.5</td>
<td>171.0</td>
<td>134.0</td>
<td>44.6</td>
<td>46.7</td>
<td>37.9</td>
</tr>
<tr>
<td>y</td>
<td>56.2</td>
<td>142.5</td>
<td>118.0</td>
<td>27.0</td>
<td>28.3</td>
<td>22.1</td>
</tr>
<tr>
<td>z</td>
<td>54.6</td>
<td>132.4</td>
<td>111.0</td>
<td>22.2</td>
<td>23.2</td>
<td>17.7</td>
</tr>
</tbody>
</table>

Procedure:
(i) For each pole, calculate the differences between the strike and the calculated azimuths.
(ii) From comparing the differences for each pole, decide which pole fits the data best.

Alternatively:
(i) Estimate the azimuth of the Euler pole at each site from the strike.
(ii) Compare the differences between the estimated and computed azimuths for each pole and decide which pole fits the data best.
RESULTS FROM EXAMPLE ON LOCATING AN EULER POLE (1)

Estimation from the variation in spreading rate

The table below lists the ratio $V/\sin D$, where $V$ is the observed spreading rate and $D$ is the angular distance from each site to the three trial Euler poles. $|\epsilon|$ is the absolute (sign ignored) deviation between $V/\sin D$ and its mean.

| site | $V/\sin D_1$ (km/Ma) | $|\epsilon_1|$ (km/Ma) | $V/\sin D_2$ (km/Ma) | $|\epsilon_2|$ (km/Ma) | $V/\sin D_3$ (km/Ma) | $|\epsilon_3|$ (km/Ma) |
|------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| a    | 114                  | 3                    | 113                  | 5                    | 115                  | 3                    |
| b    | 110                  | 1                    | 109                  | 1                    | 112                  | 0                    |
| c    | 112                  | 1                    | 110                  | 2                    | 114                  | 2                    |
| d    | 109                  | 2                    | 107                  | 1                    | 111                  | 1                    |
| e    | 107                  | 4                    | 103                  | 5                    | 107                  | 5                    |
| mean | 111                  | 2.2                  | 108                  | 3.0                  | 112                  | 2.2                  |

The spreading rate is a maximum at the Euler equator, where $D = 90^\circ$. Let $V_m$ be the maximum spreading rate. Since $\sin 90 = 1.0$, the ratio $V/\sin D = V_m$ at the Euler equator. For the correct Euler pole, $V/\sin D$ is a constant equal to $V_m$. There is nothing to choose between poles 1 and 3: both fit the data equally well. Pole 2 gives a somewhat poorer fit to the data.

Pole 3 is displaced from pole 1 in a direction roughly perpendicular to the mid-ocean ridge. This method of estimating an Euler pole is insensitive to shifts of the pole perpendicular to the direction of the ridge.
RESULTS FROM EXAMPLE ON LOCATING AN EULER POLE (2)

**Estimation from the strike directions of transform faults**

The transform faults strike approximately at right angles (90°) to the local azimuth from the site to the Euler pole. That is, the strike minus the azimuth should be 90°. This is algebraically correct if all bearings (azimuths, strikes) are measured E of N; bearings W of N must be treated as negative numbers. The table below shows the results of this subtraction.

<table>
<thead>
<tr>
<th>site</th>
<th>latitude</th>
<th>longitude</th>
<th>strike</th>
<th>Az1</th>
<th>φ − Az1</th>
<th>Az2</th>
<th>φ − Az2</th>
<th>Az3</th>
<th>φ − Az3</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>52.9</td>
<td>118.8</td>
<td>108.0</td>
<td>16.0</td>
<td>92.0</td>
<td>16.7</td>
<td>91.3</td>
<td>12.0</td>
<td>96.0</td>
</tr>
<tr>
<td>w</td>
<td>55.0</td>
<td>126.5</td>
<td>111.0</td>
<td>19.8</td>
<td>91.2</td>
<td>20.7</td>
<td>90.3</td>
<td>15.5</td>
<td>95.5</td>
</tr>
<tr>
<td>x</td>
<td>64.5</td>
<td>171.0</td>
<td>134.0</td>
<td>44.6</td>
<td>89.4</td>
<td>46.7</td>
<td>87.3</td>
<td>37.9</td>
<td>96.1</td>
</tr>
<tr>
<td>y</td>
<td>56.2</td>
<td>142.5</td>
<td>118.0</td>
<td>27.0</td>
<td>91.0</td>
<td>28.3</td>
<td>89.7</td>
<td>22.1</td>
<td>95.9</td>
</tr>
<tr>
<td>z</td>
<td>54.6</td>
<td>132.4</td>
<td>111.0</td>
<td>22.2</td>
<td>88.8</td>
<td>23.2</td>
<td>87.8</td>
<td>17.7</td>
<td>93.3</td>
</tr>
</tbody>
</table>

The average values of the strike minus the azimuth (with their absolute errors‡) are 90.5 ± 0.6°, 89.3 ± 0.7°, and 95.3 ± 0.4°. In this case the average value of strike minus the azimuth has to be close to 90° to make an acceptable fit. The averages from poles 1 and 2 both satisfy this criterion within their absolute errors. Pole 1 gives the closer fit to 90° and the smaller error. Pole 3 gives angles that are consistently too large (its small mean absolute error is a spurious effect). This method of estimating an Euler pole is insensitive to shifts of the pole along the direction from the pole to the ridge. It therefore complements the method of part 1. Geophysical modelling is not always as well constrained as this. For example, no unique density model can ever be constructed from gravity data alone. (Lecture 3 deals with gravity).

Together the spreading rates and the transform fault orientations indicate that Pole 1 (from Chase) is a slightly better fit than Pole 2 (from Fowler). Real determinations of Euler poles use many more measurements than this.

‡ The mean absolute error of the (φ − Az) values is divided by √{n − 4} = √4 = 2, where n is the number of observations, to get the absolute error of the mean.
Trigonometry: the branch of mathematics dealing with the relations of sides and angles of triangles and with relevant functions of angles.

Measurement of angles: By convention angles are measured from the positive $x$ axis, anticlockwise being the positive direction.

Consider a point $P$ on a circle of unit radius centred on the origin $O$. The convention for an angle $\theta$ is shown in the diagram below.

Circular or radian measure: many expressions in mathematics, physics and geophysics require the angle to be measured in radians.

$\theta^c = \theta$ in radians = arc length $PQ$ in the diagram below. In general

$$\theta^c = \frac{\text{arc length}}{\text{radius}}$$

$$360^\circ = 2\pi \text{ radians}, \quad 1 \text{ radian} = 57.2958^\circ$$

The superscript $c$ ($^c$) denotes circular measure or radians, just as a superscript circle ($^\circ$) denotes degrees.
Figure illustrating the measurement of angles:

2nd quadrant
$90^\circ < \theta < 180^\circ$

1st quadrant
$0^\circ < \theta < 90^\circ$

3rd quadrant
$180^\circ < \theta < 270^\circ$
or $-180^\circ < \theta < -90^\circ$

4th quadrant
$270^\circ < \theta < 360^\circ$
or $-90^\circ < \theta < 0^\circ$
APPENDIX: BASIC TRIGONOMETRY (3)

DEFINITION OF BASIC TRIGONOMETRICAL FUNCTIONS

Consider point P on the unit circle (radius OP = 1) in the previous figure.

\[
\sin \theta = \frac{RP}{OP} = RP = y \text{ coordinate of } P, \quad \csc \theta = cosec \theta = \frac{1}{\sin \theta}
\]
\[
\cos \theta = \frac{OR}{OP} = OR = x \text{ coordinate of } P, \quad \sec \theta = \frac{1}{\cos \theta}
\]
\[
\tan \theta = \frac{RP}{OR} = \frac{y \text{ coordinate of } P}{x \text{ coordinate of } P}, \quad \cot \theta = \frac{1}{\tan \theta}
\]

\sin \theta \text{ is an } \textit{odd} \text{ function: } \sin(-\theta) = -\sin \theta
\]

\cos \theta \text{ is an } \textit{even} \text{ function: } \cos(-\theta) = \cos \theta
\]

\tan \theta \text{ is an } \textit{odd} \text{ function: } \tan(-\theta) = -\tan \theta
\]

All are periodic functions, with period \(2\pi\): e.g. \(\sin(\theta + 2n\pi) = \sin \theta, \quad n \text{ integer.}\)

You should try to remember the graphs of the sine and cosine functions. These are plotted overleaf.
Graphs of the three basic trigonometrical functions
At odd multiples of $90^\circ = \pi/2$, the tangent jumps from $+\infty$ to $-\infty$ (plus infinity to minus infinity).
CAST: a mnemonic for remembering the signs of trigonometric functions.

- 2nd quadrant (S): 
  \[ \begin{array}{c}
  \text{sin} + \\
  \text{cos} - \\
  \text{tan} - 
  \end{array} \]
- 1st quadrant (A): 
  \[ \begin{array}{c}
  \text{all} + 
  \end{array} \]
- 3rd quadrant (T): 
  \[ \begin{array}{c}
  \text{sin} - \\
  \text{cos} - \\
  \text{tan} + 
  \end{array} \]
- 4th quadrant (C): 
  \[ \begin{array}{c}
  \text{sin} - \\
  \text{cos} + \\
  \text{tan} - 
  \end{array} \]
APPENDIX: BASIC TRIGONOMETRY (6)

TRIGONOMETRIC IDENTITIES

Fundamental identities:

\[
\tan \theta = \frac{\sin \theta}{\cos \theta}
\]

\[
\sin^2 \theta + \cos^2 \theta = 1
\]

\[
\sin(A + B) = \sin A \cos B + \cos A \sin B
\]

\[
\cos(A + B) = \cos A \cos B - \sin A \sin B
\]

From these others can be derived, such as:

\[
\sin(A - B) = \sin A \cos B - \cos A \sin B
\]

\[
\cos(A - B) = \cos A \cos B + \sin A \sin B
\]

Refer to a mathematics text if you want to pursue others!