1113 CONTRACTS AND GAME THEORY
Midterm Exam
Universitat Pompeu Fabra – Spring 1997
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1. A strategy profile σ^* is a *strong* equilibrium if there is no other strategy profile σ' such that $u_j(\sigma') > u_j(\sigma^*)$ for all j with $\sigma'_j \neq \sigma^*_j$.

   (a) (15) Is the set of Nash equilibria a subset of the set of strong equilibria or a superset? Why?

   (b) (20) Show that if all the Nash equilibria of a game give rise to *strictly* Pareto-inferior outcomes (outcomes where all players are *strictly* worse off than under some alternative) the game has no strong equilibria. Give an example of such game.

   (c) (15) Give an example of a game which has at least one strong equilibrium, and where the sets of strong equilibria and Nash equilibria are different.

2. Consider the extensive-form game depicted in the following figure.

   (a) (10) Write the strategic form of this game.

   (b) (5) Find the strictly dominated strategies of this game.

   (c) (15) Show that there is no Nash equilibrium with $\sigma_1(b) = 0$ (You can do it by contradiction, show that if in equilibrium $\sigma_1(b) = 0$ then $\sigma_2(df) = 0$ and use that fact to obtain a contradiction).

   (d) (15) Show that if $\sigma_1(b) > 0$ then $\sigma_2(ce) = 0$ and use that fact to show that with the remaining strategies there is a unique Nash equilibrium (in mixed strategies) for this game.

   (e) (5) What are the subgame perfect equilibria in this game?

3. Two players have to announce (simultaneously and independently) an integer number. The player who announces the largest integer gets a prize of \$100. The player who announces the lowest integer gets nothing. If both announce the same integer they get \$50 each.

   (a) (10) Show that this game has no equilibrium in pure strategies

   (b) (10) Show that if an agent i_1 uses a mixed strategy σ_{i_1} such that for some integer j, $\sigma_{i_1}(k) = 0$ for $k \geq j$ (that is, there is a maximal integer that receives positive weight under σ_{i_1}) then $i_2 \neq i_1$ has a strategy that guarantees that i_2 wins the prize.

   (c) (15) Show that if an agent i_1 uses a strategy σ_{i_1} such that for any integer j there is some $k \geq j$ with $\sigma_{i_1}(k) > 0$ (that is, there is no maximal integer that receives positive weight under σ_{i_1}) there is no best response for the opponent (that is, for all strategies σ_{i_2} there is some σ'_{i_2} with $u_{i_2}(\sigma'_{i_2}, \sigma_{i_1}) > u_{i_2}(\sigma_{i_2}, \sigma_{i_1})$.

   (d) (15) Use the previous two answers to show that this game has no Nash equilibrium in mixed strategies either. Which assumption of the existence theorems we studied is not satisfied in this game?

4. Players 1, 2 and 3 are voting in a committee to choose among three options, called α, β and γ. First, each player submits a secret vote for one of the three options. Then the votes are opened. If any option gets two votes, then it is the outcome of the game. Otherwise, if there is a (necessarily three-wise) tie, then player 1 (who is the chairperson of the committee) will choose the outcome. The players' payoffs depend on the outcome and are shown in the table that follows,

	Player		
Option	1	2	3
α	8	0	4
β	4	8	0
γ	0	4	8

   (a) (5) How many strategies does player 1 have? And the other players?

   (b) (10) Show that the strategies for player 1 that involve choosing either β or γ after a tie are weakly dominated.

   (c) (15) Show that the strategies for player 1 that involve choosing either β or γ in the first stage (and then choosing α after a tie) are weakly dominated.

   (d) (15) Show that after eliminating the strategies for player 1 in the previous questions, the strategy α is weakly dominated for player 2 and strategy α and β are weakly dominated for player 3.

   (e) (5) Show that after the previous rounds of deletion one of the two surviving strategies for player 2 is strictly dominated. What is the (unique) surviving strategy profile?