MICROECONOMICS II
Problem set 2
Universitat Pompeu Fabra – Winter 2006
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1. At the beginning of this game, players 1 and 2 each put a dollar in the pot. Next, player 1 draws a card from a shuffled deck in which half the cards are red and half are black. Player 1 looks at his card privately and decides whether to raise or fold. If player 1 folds then he shows the card to player 2 and the game ends; in this case, player 1 takes the money in the pot if the card is red, but player 2 takes the money in the pot if the card is black. If player 1 raises, the he adds another dollar to the pot and player 2 must decide whether to pass or meet. If player 2 passes, then the game ends and player 1 takes the money in the pot. If player 2 meets, then she also must add another dollar to the pot, and then player 1 shows the card to player 2 and the game ends; in this case, again, player 1 takes the money in the pot if the card is red, and player 2 takes the money in the pot if the card is black.

   (a) Write the extensive form and the normal form for this game and compute its Nash and subgame perfect equilibria.

2. An individual consumes in two periods and has an initial endowment of capital, k_1. A government levies taxes to pay for the vacations of its president in the *Coto de Doñana*.

   The utility of the individual is $u_e(c_1, c_2) = \ln c_1 + (1+\theta)^{-1} \ln c_2$ and the utility of the government is $u_g(c_2, g_2) = \ln g_2 + \epsilon \ln c_2$, where c_i is consumption of the individual in period i, and g_2 is government spending.

   The production function is linear with

$$c_1 + k_2 = Rk_1 \tag{1}$$

$$c_2 + g_2 = Rk_2. \tag{2}$$

   In period 1 the individual decides c_1 and k_2 subject to equation (**??**). In period 2 the government announces a tax rate on capital τ_k, $0 \leq \tau_k \leq R$. Government spending will be $g_2 = \tau_k k_2$ and c_2 will satisfy equation (**??**)

(a) Find the pure strategy subgame perfect equilibrium of this game.

(b) Assume that the timing is different and the government can announce tax rates before period 1 starts which he cannot change. What is the subgame perfect equilibrium now? Do agents have larger equilibrium payoffs in this game or in the previous one?

3. Let Γ be an extensive form game with perfect information, with no ties in payoffs. Show that the process of iterative deletion of weakly dominated strategies on the normal form game derived from Γ gives rise to a unique Nash equilibrium payoff profile, which coincides with the one that results from solving Γ by backward induction.

4. Let G_2 be a game in extensive form with imperfect information in which there are no moves of Nature. Assume that G_1 differs from G_2 only in that one information set of player 1 in G_2 is split into two information sets in G_1. Show that all Nash equilibria in pure strategies of G_2 correspond to Nash equilibria of G_1. Show that the requirement that there be no moves of Nature is essential for this result.

5. Find the Nash and subgame perfect equilibria for the following game: