MICROECONOMICS II
Problem set 3
Universitat Pompeu Fabra – Winter 2004
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1. Assume we have the following game:

	α_2	β_2
α_1	X, X	$X, 0$
β_1	$0, X$	$8, 8$

where X is a random variable which takes values in the set $V = \{9, 8, 7, 6, 5\}$ with equal probability for all values in V. At the beginning of the game each of the two players receives independently a signal $\sigma_i \in \Sigma = \{T, U, W, Y, Z\}$ about the value of X. The following table explains how signals are related to the value of X.

Value of X	9	8	7	6	5
Signal	T or U	T or U or W	U or W or Y	W or Y or Z	Y or Z

Given a value of X, each one of the possible hints for that value is equally probable.

   (a) What is the expected value of playing strategy s_i given a signal σ_i?

   (b) Does an agent who receives a signal $\sigma_i = T$ have a strictly dominant strategy? What about an agent with signal $\sigma_i = U$?

   (c) Is some strategy dominated for agents with other signals, (perhaps after the agents with $\sigma_i = T$ and $\sigma_i = U$ eliminate some strategy)?

   (d) What are the Nash equilbria of this game?

2. Two players must jointly produce a certain public good. To this end, each of them must contribute some amount of labor input $l_i \in [0, 1]$. The productivity of individual 2 is common knowledge, whereas that of individual 1 is private information (i.e. it is known only to herself). *A priori*, this productivity can be either *high* with probability p or *low* with probability $(1-p)$. Both individuals decide simultaneously how much to contribute to the production of the public good. Once their decisions have been made, the output produced is as given by the following production function:

$$y(l_1, l_2) = \begin{cases} \sqrt{2l_1 + l_2} \text{ if individual 1 is highly productive} \\ \sqrt{l_1 + l_2} \text{ otherwise} \end{cases}$$

Given the individuals' labor contribution and the induced production of the public good, each individual $i \in \{1, 2\}$ obtains a utility (i.e. payoff) given by the function $U_i(l_i, y) = (1 - l_i)y^2$.

   (a) Formalize the situation as a Bayesian game.

   (b) Compute the Bayesian Nash equilibria of the game.

   (c) Determine the effect of an increase in p on the labor contributions decided at equilibrium.

3. Two players bid $x_i \geq 0$, for an object worth 1 euro to both of them. The object goes to the highest bidder, *both* players pay the lowest bid. Thus

$$u_i(x_1, x_2) = \begin{cases} 1 - x_j, \text{if} \quad x_i > x_j \\ 1/2 - x_i, \text{if} \quad x_i = x_j \\ -x_i, \text{if} \quad x_i < x_j \end{cases}$$

   (a) What are the pure-strategy Nash-equilibria for this game?

4. With equal probabilities Player 1 is dealt a card H or a card L. Player 2 is never dealt a card, and does not get to look at the card of Player 1 until the end of the game. After looking at his card Player 1 decides to either *Play* or *Fold*. If he *Folds*, the game ends, he loses 1 euro and Player 2 wins 1 euro. If he *Plays*, Player 2 must now consider whether to *Play* or *Fold*. If he decides to *Fold*, then he loses 1 euro and Player 1 wins 1 euro. If he *Plays*, then the card is shown. If it is H, Player 1 wins 4 euros, and Player 2 loses 4 euros. If it is L, Player 1 loses 4 euros, and Player 2 wins 4 euros.

   (a) Draw a game tree and the strategic form of this game.
   (b) Show that player 1 has only 2 undominated strategies.
   (c) Find the Bayes-Nash equilibria of this game.