1. A government and an agent are involved in the following strategic context. The agent must choose an action a from the set $A = \{0, 1\}$. The government would like to influence the agent's choice. To try to do so, the government publicly announces, *before* the agent selects her action, a monetary transfer rule $t : A \to \Re$ that is to be automatically implemented after the agent has made her decision. Assume that the monetary transfers induced by t (i.e., the values of $t(a)$, for each $a \in A$) can take only two values: zero and a certain fixed positive value, normalized to one. Let the objective of the government be to maximize $U_g(a, t) \equiv 2a - t$ and that of the agent to maximize $U_a(a, t) \equiv t - c(a)$, where $c(a)$ is the (monetary) cost of her action. Furthermore, postulate that $c(0) = 0$ and $c(1) = 1/2$.

   (a) Represent the game in extensive form under the assumption that the transfer rule cannot depend on the agent's choice.

   (b) Represent the game in extensive form when the government can choose a transfer rule that depends on the agent's choice.

   (c) Define, for each of the two scenarios considered in (a) and (b) the strategy sets of each player (government and agent) and represent the game in strategic form.

   (d) Find, for both scenarios, the Nash and subgame-perfect equilibria and discuss their salient features.

2. Let s be a strategy profile in an extensive game with perfect information Γ; suppose that $i(h) = i$, $s_i(h) = a$, and $a' \in A(h)$ with $a' \neq a$. Consider the game Γ' obtained from Γ by deleting all histories of the form (h, a', h') for some sequence of actions h', and let s' be the strategy profile in Γ' that is induced by s. Show that if s is a subgame-perfect equilibrium of Γ then s' is a subgame-perfect equilibrium of Γ'.

3. Armies 1 and 2 are fighting over an island initially held by a battalion of army 2. Army 1 has K battalions and army 2 has L. Whenever the island is occupied by one army the opposing army can launch an attack. The outcome of the attack is that the occupying battalion and one of the attacking battalions are destroyed; the attaking army wins and, so long as it has battalions left, occupies the island with one battalion. The commander of each army is interested in maximizing the number of surviving battalions but also regards the occupation of the island as worth more than one battalion but less than two. (If after an attack, neither army has battalions left, then the payoff of each commander is 0.)

   (a) Draw the extensive form for this game, for $(K, L) = (2, 2)$ and also for $(2, 3)$ and find their subgame-perfect equilibria.

(b) What will the subgame-perfect outcome be in general, as a function of K and L, and why is this subgame-perfect?

4. Consider the sequential game of Fig. 1. In the first stage player 1 chooses between a certain payoff of 2 or a simultaneous-moves game with player 2 with payoffs indicated in the figure (player 1's payoffs to the left).

   (a) Find the set of pure-strategy Nash equilibria of the game.

   (b) Find the set of pure-strategy subgame-perfect equilibria of the game.

   (c) Find the set of pure-strategy subgame-perfect equilibria that survive iterated deletion of weakly dominated strategies.
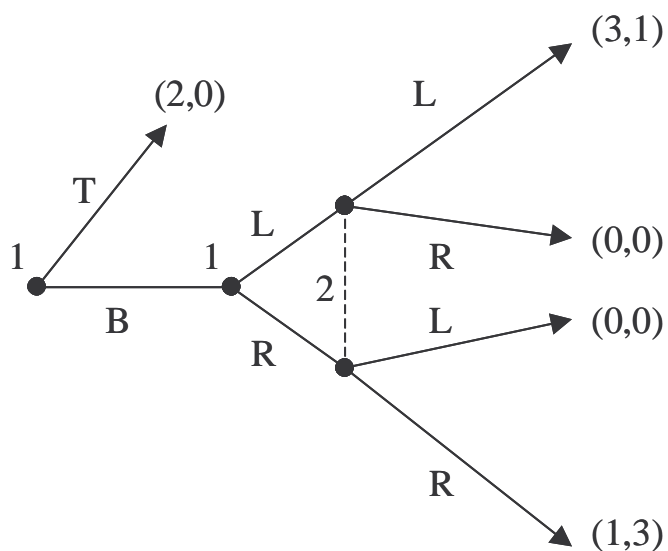

Fig. 1