Return Migration: Theory and Empirical Evidence from the UK, Appendix

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1 Introduction

From an economic point of view, migration is part of the general tendency of individuals to seek a higher standard of living. Different economies provide different economic opportunities and, when possible, migrants flow from poor to rich countries. Some immigrants return to their country of origin, where they can put to a better use the new skills acquired abroad, but most stay at the new country. There, they follow a gradual process of economic assimilation, reflected in wage increase in occupational mobility. However, this process differs for immigrants with different attributes and depends on market condition as well as the immigration policies of the receiving countries. The purpose of this paper is to describe, theoretically and empirically, some of the determinants of economic assimilation. How well immigrants do in the new countries naturally concerns those who moved, as well as prospective immigrants, but the issue is much broader because the rate of assimilation of immigrants affects whether they complement or substitute local workers. Immigrants often work, initially, in simple jobs and may compete with unskilled local workers but complement local skilled workers, while later as they assimilate they may also compete with local skilled workers. Another perspective that we shall not discuss here is the impact on the countries of origin who may suffer an economic loss (e.g., a brain drain).

2 Human capital, skills and immigration

2.1 Assumptions

Human capital is an aggregate that summarizes individual skills in terms of productive capacity. Different skills are rewarded differentially in different countries.

We assume the connection between individual skills and productive capacity may be represented as

$$ln K_j = \sum_s \theta_{sj} S_s,$$
(1)

where K_j is the productive capacity of a person if he works in country j, S_s is the quantity of skill s possessed by the individual and θ_{sj} is a non-negative parameter that represent the contribution of skill s to production in country j. To simplify the exposition, we consider the case of only two countries, the receiving country and the country of origin, and two skills.

Skills are initially endowed and can then be augmented by acquiring work experience. We consider here a "learning by doing" technology, whereby work in country j augments skill s by γ_{sj} per unit of time worked. Note the joint production feature of this technology. Working in any one country j can influence skills that are useful in other countries. Yet, such experience may be more relevant to some particular skills. In this way, we obtain that work experience is transferable but not necessarily general. We assume that skill 1 is more valuable than skill 2 in the receiving country while skill 2 is more valuable than skill 1 in the country of origin. That is,

$$\theta_{11} > \theta_{12},
\theta_{22} > \theta_{21}.$$
(2)

We shall also assume that skill 1 is accumulated at a faster rate than skill 2 in the receiving country while skill 2 is accumulated more quickly than skill 1 in the country of origin.

$$\gamma_{11} > \gamma_{21},
\gamma_{22} > \gamma_{12}.$$
(3)

Together, these two assumptions distinguish the two countries in terms of the skills that used and generated there. Think of country 1 to be more modern (developed) than country and suppose that skill 1 represents "managerial" or "intellectual" skills and skill 2 represents "work" or "physical" skills. Then, one may expect that managerial skills have a higher relative price in the developed country and also that work experience in the developed country will augment theses at a higher rate. In contrast, "physical" skills may be more valuable in country 2 and also augmented there at a faster rate.

We assume that γ_{ij} and θ_{ij} are all constant parameters and that time flows continuously. Then, a person who works in country 1 accumulates local human capital at a rate

$$\frac{\dot{K}_1}{K_1} = \theta_{11}\gamma_{11} + \theta_{21}\gamma_{21} \equiv g_{11} \tag{4}$$

and foreign human capital at a rate

$$\frac{\dot{K}_2}{K_2} = \theta_{12}\gamma_{11} + \theta_{22}\gamma_{21} \equiv g_{21}. \tag{5}$$

Similarly, a person who works in country 2 accumulates local human capital at a rate

$$\frac{\dot{K}_2}{K_2} = \theta_{12}\gamma_{12} + \theta_{22}\gamma_{22} \equiv g_{22} \tag{6}$$

and a foreign human capital at a rate

$$\frac{\dot{K}_1}{K_1} = \theta_{11}\gamma_{12} + \theta_{21}\gamma_{22} \equiv g_{12}.\tag{7}$$

As seen, the growth in local and foreign human capital for workers in each of the two countries depends on both the prices and learning rates of the two skills. Because prices of skills and the learning rates of each skill differ across countries the rate of local and foreign capital can differ too. A crucial feature for our analysis is the transferability of experience across countries. In this paper, we shall distinguish between to two basic cases:

- 1) Partial transferability of experience, $g_{11} > g_{21}, g_{22} > g_{12}$, which means that work experience in any given country has a larger impact on the accumulation of local human capital than on foreign human capital.
- 2) **Super transferability,** which means that work experience acquired in some country has a larger impact on the accumulation of foreign human capital than on local human capital. This property is usually not symmetrical and characterizes learning centers. For concreteness, we shall consider the case that country 1 is such a center and then $g_{21} > g_{11}, g_{22} > g_{12}$

Note that these definitions do not apply directly to transferability of skills or productive capacity but rather to the role of experience in different countries to augment skills that have different values in different countries. In particular, even though an immigrant from a developing country acquires more managerial skills in the receiving country (because $\gamma_{11} > \gamma_{21}$) the work skills that are acquired are more valuable in the home country, so it is possible that human capital applicable to the home country, K_1 , grows faster than the human capital that is applicable to the receiving country, K_2 .¹

Firms in each country reward individual skills indirectly by renting human capital at the market-determined rental rate, R_j , implying that a worker with

$$g_{11} > g_{21} \iff \frac{\theta_{11} - \theta_{12}}{\theta_{22} - \theta_{21}} > \frac{\gamma_{21}}{\gamma_{11}},$$

$$g_{22} > g_{12} \iff \frac{\theta_{11} - \theta_{12}}{\theta_{22} - \theta_{21}} < \frac{\gamma_{22}}{\gamma_{12}}.$$

Thus, partial transferability requires

$$\frac{\gamma_{22}}{\gamma_{12}} > \frac{\theta_{11} - \theta_{12}}{\theta_{22} - \theta_{21}} > \frac{\gamma_{21}}{\gamma_{11}},$$

and super transferability requires

$$\frac{\gamma_{22}}{\gamma_{12}} > \frac{\gamma_{21}}{\gamma_{11}} > \frac{\theta_{11} - \theta_{12}}{\theta_{22} - \theta_{21}}.$$

¹In terms of the basic parameters,

a given bundle of skills earns in country j at time t

$$Y_j(t) = R_j \exp(\sum_s \theta_{sj} S_s(t)). \tag{8}$$

Thus, the parameter θ_{sj} is proportional to the increase in earning capacity associated with a unit increase in skill x_s if the individual works in country j. Having assumed that θ_{sj} is independent of the quantity of skill s possessed by the individual, these coefficients may be viewed as the implicit "price" (or "rate of return") of skill s in country j. We shall assume that the rental rate for human capital in the receiving country, R_1 exceeds the rental rate that human capital receives in country 2, R_2 . This difference in rental rates can be sustained because immigration into the receiving country as regulated and only some of those who wish to enter are allowed in.

For several reasons, it is likely that immigrants who enter the receiving country do not immediately receive the same rental for their human capital as natives. First, it takes time for immigrants to find a suitable job that matches their skills in the receiving country. Second, employers may be uncertain of the immigrants' quality and may update their beliefs based on observed performance. Finally, immigrants may need time to learn the local language and labor market institutions. To describe these processes, we adopt the following functional form

$$R_{1m}(t-\tau) = e^{-\lambda((t-\tau))}R_2 + (1 - e^{-\lambda((t-\tau))})R_1,$$
(9)

where τ is the time of entry into the new country. That is, the rental rate that an immigrant from country 1 receives in country 2 is a weighted average of the rental rate in the country of origin, R_2 , and the rental rate in the receiving country R_1 . Initially, immigrants receive the same rental rate as abroad, R_2 , but as they spend more time in the host country, the rental rate rises and approaches the rental rate of natives, R_1 . The parameter $\lambda > 0$ controls the speed of adjustment given by

$$\dot{R}_{1m} = \lambda [R_1 - R_2] e^{-\lambda((t-\tau))} = \lambda (R_1 - R_{1m}(t-\tau)). \tag{10}$$

With this specification, the gap in the rentals rate of natives and immigrants narrows at a decreasing rate.

The rental rate that immigrants receive for their human capital is only one consideration in determining the gap in wages between natives and immigrants. Another consideration is the difference in the composition of skills of the two groups. Immigrants are not a random sample of the population and to understand the time pattern of the average wage gap between natives and immigrants we need to analyze the migration decisions of individuals with different skills, which depend on how skills are valued and at what rate they are augmented in the two countries. For this purpose, we shall assume that individual planning horizon (work period) is sufficiently long and the interest rate is sufficiently high to allow us to use infinite horizon as a valid approximation. In particular, we shall assume that all the growth rates of human capital in any country fall short of the fixed interest rate, r.

2.2 Analysis

We wish to address the following three questions:

- Who shall immigrate from country 2 to country 1 and when?
- Which of these immigrants will return to country 1, if at all, and when?
- What are the implications of selection to the observed rate of assimilation of immigrants in the receiving country?

We show that

Proposition 1 If work experience in any given country is partially transferable then any person who wishes to migrate from country 2 to country 1 will aim to do so as early as possible and, if allowed to enter, he will not reverse his decision later it later and return to his home country.

Proposition 2 If experience accumulated in country 1 is super transferable, that is, a person working there accumulates foreign human capital at a faster rate than local human capital then any person who wishes to migrate from country 2 to country 1 will do it immediately (as in the previous case) but, in this case, he will later return to his home country after a finite period of time.

We conduct the analysis in the steps. We first examine the return decision of an immigrant who is already in the receiving country and investigate who shall return and when. Based on the results of this second stage, we show that immigration is never postponed and discuss who shall emigrate.

Imagine an immigrant who moved from country 2 to 1 at time τ and considers whether to return to the home country at time ε . Conditional on entry at τ , the present value (evaluated at the time of entry τ) of staying at country 1 for a period $\tau - \varepsilon$ and then moving back at time ε is

$$V(\varepsilon,\tau) = K_{1}(\tau) \int_{\tau}^{\varepsilon} e^{-r(t-\tau)+g_{11}(t-\tau)} R_{1m}(t-\tau) dt$$

$$+R_{2}K_{2}(\tau) e^{g_{21}(\varepsilon-\tau)} \int_{\varepsilon}^{\infty} e^{-r(t-\tau)+g_{22}(t-\varepsilon)} dt$$

$$= K_{1}(\tau) \int_{0}^{\varepsilon-\tau} e^{(g_{11}-r)x} R_{1m}(x) dx + \frac{R_{2}K_{2}(\tau)}{r-g_{22}} e^{(g_{21}-r)(\varepsilon-\tau)},$$
(11)

where $K_1(\tau)$ and $K_2(\tau)$ are the amount of local human and foreign human capital that the immigrant brought from abroad upon arrival at τ .

Differentiating with respect to ε we get

$$V_{\varepsilon}(\varepsilon,\tau) = K_1(\tau)e^{(g_{11}-r)(\varepsilon-\tau)}R_{1m}(\varepsilon-\tau) + (g_{21}-r)\frac{R_2K_2(\tau)}{r - g_{22}}e^{(g_{21}-r)(\varepsilon-\tau)}$$
(12)

The first term on the RHS of (12) is the marginal gain from staying in the receiving country and postponing the return to the country, in terms of the additional earnings generated in the receiving country. The second term is marginal cost of extending the stay in the receiving country and postponing the return to the home country, in terms of foregone potential. Now consider the time in which the marginal gains from extending the stay just equal the marginal costs. If such a time exists, it must satisfy the condition $V_{\varepsilon}(\varepsilon, \tau) = 0$, which can be rewritten as

$$\frac{R_{1m}(\varepsilon - \tau)}{R_2} = \frac{r - g_{21}}{r - g_{22}} \frac{K_2(\tau)}{K_1(\tau)} e^{(g_{21} - g_{11})(\varepsilon - \tau)}$$
(13)

The left hand side of equation () is the ratio of the rental rates of human capital that immigrants obtain in the in the receiving country and in their the home country, respectively. Recall that, by assumption, $R_{1m}(\varepsilon - \tau)$ an increasing concave function that starts at R_2 upon entry $(\varepsilon - \tau = 0)$ and approaches R_1 as $\varepsilon - \tau$ approaches infinity. The ratio $\Omega(\tau) \equiv \frac{K_2(\tau)}{K_1(\tau)}$ is a measure of the relative earning capacity of the immigrants in the two countries upon arrival, at which point the rental rates that the immigrant receives is the same in the two countries. When the immigrant stays in the receiving country this ratio changes with time spent in the new country and its time pattern is given by

$$\Omega(\varepsilon - \tau) = \Omega(\tau)e^{(g_{21} - g_{11})(\varepsilon - \tau)}.$$
(14)

This function is increasing if $g_{21} > g_{11}$, decreasing if $g_{11} > g_{21}$ and convex in both cases. The ratio $\frac{r-g_{21}}{r-g_{22}}$ is a conversion factor that translates prospective income streams into present values. Thus, $\frac{R_2K_2(\tau)}{r-g_{22}}$ is the present value of the exponentially growing earning stream that the immigrant would obtain abroad and a $\frac{R_2K_1(\tau)}{r-g_{21}}$ is the present value of the exponentially growing earning stream that the immigrant would obtain in the receiving country if his rental rate would remain constant at R_2 .

Figure 1 shows the determination of ε when $g_{11}>g_{21}$. The rising curve represent $\frac{R_{1m}(\varepsilon-\tau)}{R_2}$ and the declining curve describes $\frac{r-g_{21}}{r-g_{22}}\frac{K_2(\tau)}{K_1(\tau)}e^{(g_{21}-g_{11})(\varepsilon-\tau)}$. The two curves intersect at the point in which $V_{\varepsilon}(\varepsilon,\tau)=0$. However, it is seen in the graph that moving bit right to the of intersection $\frac{R_{1m}(\varepsilon-\tau)}{R_2}>\frac{r-g_{21}}{r-g_{22}}\frac{K_2(\tau)}{K_1(\tau)}e^{(g_{21}-g_{11})(\varepsilon-\tau)}$, which implies that the $V_{\varepsilon}(\varepsilon,\tau)>0$ and the gain from postponing the return exceeds the cost so that and the individual wants to postpone his return even further. By a similar argument, we see that moving a bit left the gains from leaving even earlier rise. he want to leave sooner. Therefore, there is no interior optimal solution satisfying $V_{\varepsilon}(\varepsilon,\tau)=0$ and $V_{\varepsilon\varepsilon}(\varepsilon,\tau)<0$, implying that the immigrant will either stay for ever in the receiving country or return immediately to the home county. This outcome is a consequence of the learning by doing process, If the immigrant ever finds it valuable to stay then because his human capital in the receiving country rises faster than his human capital at the home country, $g_{11}>g_{21}$, staying a bit more makes the option of staying in the receiving country look even better.

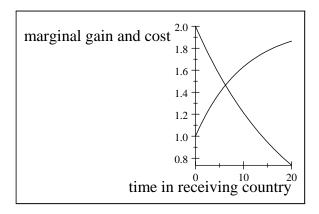
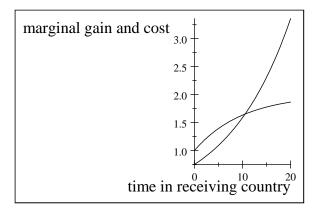


Figure 2 shows the determination of ε when $g_{21} > g_{11}$. In this case if an intersection exists it satisfies the second order conditions. Moving a bit right to the of intersection, the marginal gain from postponing is smaller than the marginal cost and the individual wants to leave earlier. Moving a bit left he want to leave later. Therefore, an optimal solution may exist such that an immigrant who chose to enter the receiving country will later wish to leave because his human capita that is applicable at the home country rises in the receiving contract at a faster rate than the human capital that is applicable if he stays at the receiving country. Because these gains can only materialize by actually moving back to the home country, the immigrant will always return after a finite period of time. Examining equation (12) and the figure it is seen that the immigrant will leave sooner after his arrival (i.e., $\varepsilon - \tau$ declines) if $\Omega(\tau)$ rises, because the forgone earnings at home while learning abroad are higher. For sufficiently high $\Omega(\tau)$ he will leave immediately which means that he should have never come.



As a consequence of the infinite horizon approximation, the time that an immigrants will spend in the receiving country $\varepsilon - \tau$ depends only on the ratio

 $\Omega(\tau) = \frac{K_2(\tau)}{K_1(\tau)}$ that the immigrant had upon entry and not on the chronological time of entry itself. We can therefore write the *optimized* value of $V(\varepsilon, \tau)$ of those who stay for a while in the form

$$\underset{\varepsilon}{MaxV}(\varepsilon,\tau) = K_1(\tau)H(\Omega(\tau)) \tag{15}$$

where, by the envelope theorem,

$$H'(\Omega(\tau)) = \frac{R_2}{r - g_{22}} e^{(g_{21} - r)(\varepsilon - \tau)} > 0.$$
(16)

Due to our result that $\varepsilon - \tau$ declines with $\Omega(\tau)$ (and given that $g_{21} < r$), $H''(\Omega(\tau)) > 0$.

Consider now the determination of the time of entry τ . Let us begin with the simpler case in which experience is partially transferable, $g_{11} > g_{21}$. We have shown above that in such a case an immigrant will either stay for ever or leave immediately. We only need to discuss the problem of those who chose to stay conditioned on τ . The present value of life time earnings (evaluated at time 0) of an migrant who moves from the home country to the receiving country at time at time τ is

$$W(\tau) = R_2 K_2(0) \int_0^{\tau} e^{(g_{22} - r)t} dt + K_1(\tau) \int_{\tau}^{\infty} e^{-rt + g_{11}(t - \tau)} R_{1m}(t - \tau) dt$$
(17)
$$= R_2 K_2(0) \int_0^{\tau} e^{(g_{22} - r)t} dt + K_1(0) e^{(g_{12} - r)\tau} \int_0^{\infty} e^{(g_{11} - r)x} R_{1m}(x) dx$$

Thus

$$W'(\tau) = e^{-rt} \{ R_2 K_2(0) e^{g_{22}\tau} + (g_{12} - r) e^{g_{12}\tau} K_1(0) \int_0^\infty e^{(g_{11} - r)x} R_{1m}(x) dx \},$$
(18)

$$W''(\tau) = -rW'(\tau) + e^{-rt} \{ (g_{22}R_2K_2(0)e^{g_{22}\tau} + (g_{12} - r)g_{12}e^{g_{12}\tau}K_1(0) \int_0^\infty e^{(g_{11} - r)x}R_{1m}(x)dx \}.$$

Evaluating $W''(\tau)$ at a point satisfying $W'(\tau) = 0$, we obtain

$$W''(\tau) = e^{-r\tau} \{ (g_{22} - r) R_2 K_2(0) e^{g_{22}t} - (g_{12} - r) R_2 K_2(0) e^{g_{22}t} \}$$
(19)
= $R_2 K_2(0) e^{(g_{22} - r)t} (g_{22} - g_{12}) > 0.$

Implying that there is *no* interior solution for τ and a potential immigrant would choose either to move immediately or not move at all.

A slightly more difficult case arises if $g_{21} > g_{11}$. In this case too, we only need to deal with those who wish to stay, conditioned on τ , but we now must

account for the fact that such immigrants will return back to the home at some endogenously determined time ε . In this case, the present value of life time earnings (evaluated at time 0) is

$$W(\tau) = R_2 K_2(0) \int_0^{\tau} e^{(g_{22} - r)t} dt + e^{-r\tau} K_1(\tau) H(\Omega(\tau)), \tag{20}$$

where we assume that the immigrant will optimize his time of return. Recall that, by definition, $K_1(\tau) = K_1(0)e^{g_{12}\tau}$, $K_2(\tau) = K_2(0)e^{g_{22}\tau}$ and $\Omega(\tau) = \frac{K_2(\tau)}{K_1(\tau)} = \frac{K_2(0)}{K_1(0)}e^{(g_{22}-g_{12})\tau}$. Therefore,

$$W'(\tau) = e^{-r\tau} \{ R_2 K_2(\tau) + K_1(\tau) g_{12} H(\Omega(\tau)) + (g_{22} - g_{12}) K_2(\tau) H'(\Omega(\tau)) \}$$

$$W''(\tau) = -rW'(\tau) + e^{-r\tau} \{ (g_{22} R_2 K_2(\tau) + K_1(\tau) (g_{12})^2 H(\Omega(\tau)) + H'(\Omega(\tau)) (g_{22} - g_{21}) [g_{12} K_1(\tau) + g_{22} K_2(\tau)] + (g_{22} - g_{12})^2 K_2(\tau) H''(\Omega(\tau)) \}$$

$$(21)$$

Evaluating $W''(\tau)$ at a point satisfying $W'(\tau) = 0$, we obtain

$$W''(\tau) = e^{-r\tau} \{ (g_{22}R_2K_2(\tau) + g_{12}[(g_{12} - g_{22})K_2(\tau)H'(\Omega(\tau)) - R_2K_2(\tau)] + H'(\Omega(\tau))(g_{22} - g_{12})[g_{12}K_1(\tau) + g_{22}K_2(\tau)]) + (g_{22} - g_{12})^2K_2(\tau)H''(\Omega(\tau)) \}$$

$$= e^{-r\tau} \{ R_2K_2(\tau)(g_{22} - g_{12}) + (g_{22} - g_{12})H'(\Omega(\tau))[g_{22}K_2(\tau) + g_{12}K_1(\tau) - g_{12}K_2(\tau)] + (g_{22} - g_{12})^2K_2(\tau)H''(\Omega(\tau)) \}.$$
(22)

Because we assume now that $g_{21} > g_{11}$ and, therefore, $H''(\Omega(\tau) > 0$, as we have shown. As in the previous case, we maintain the assumption that that $g_{22} > g_{12}$. We conclude that It then follows that $W'(\tau) = 0 \Longrightarrow W''(\tau) > 0$ so that there is no optimal departure time and a potential immigrant would either leave immediately or stay in the home country for ever.

We can now complete the analysis by addressing the question who will migrate and who will return. Again we begin with the simpler case in which case $g_{11} > g_{21}$. We have shown above that in such a case an immigrant will either stay for ever or leave immediately. The choice between these two alternatives is reduced to a comparison of the potential life time earnings in the two countries and a person will wish to emigrate to the receiving country immediately if

$$R_2 K_2(0) \int_0^\infty e^{(g_{22}-r)t} dt < K_1(0) \int_0^\infty e^{(g_{11}-r)t} R_{1m}(t) dt.$$
 (23)

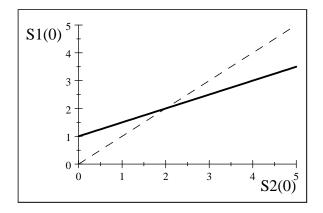
This comparison of present values can also be written in the form

$$\Omega(0) < \left[\frac{r - g_{22}}{r - g_{11}} - \frac{r - g_{22}}{r + \lambda - g_{11}}\right] \frac{R_1}{R_2} + \frac{r - g_{22}}{r + \lambda - g_{11}} = T$$
 (24)

Thus, there is some critical value of $\Omega(0)$ that triggers emigration. We can further reduce this relationship and write

$$(\theta_{11} - \theta_{12})S_1(0) + (\theta_{12} - \theta_{22})S_2(0) > -\ln T$$
(25)

Different individuals have different skills and the set of people that wish to migrate is all those whose bundle of initial skills (a pair $S_1(0), S_2(0)$) places them above the solid line described in Figure 3



Because skill 1 has higher value in country 1 individuals with relatively higher endowment of that skill are more likely to emigrate. Assuming that the distribution of skills is the same in the two countries and that immigrants that ate allowed into the receiving country are a random sample of those who apply, immigrants who enter the receiving country have a higher endowment than natives of the more highly valued skill 1. Initially, immigrants will receive lower wages than natives because the rental rate of human capita that they receive is lower than that of natives, $R_2 < R_1$. However, as the rental rate that immigrants receive in the new country rises and approaches R_1 , they will eventually overtake the natives in terms of average wages.

The situation is quite similar if $g_{21} > g_{11}$. A person in the home country will wish to emigrate to the receiving country immediately if

$$R_2 K_2(0) \int_{0}^{\infty} e^{(g_{22} - r)t} dt < K_1(0) H(\Omega(0))$$
(26)

or

$$R_2\Omega(0)\int_0^\infty e^{(g_{22}-r)t}dt < H(\Omega(0))$$
(27)

The derivative of the LHS of (0) with respect to $\Omega(0)$ is $R_2 \int\limits_0^\infty e^{(g_{22}-r)t} = \frac{R_2}{r-g_{22}}$

while, by (16), the derivative with respect to RHS is $\frac{R_2}{r-g_{22}}e^{i(g_{21}-r)(\varepsilon-\tau)}$, which is smaller. Thus, as in the previous case, there is a unique $\Omega(0)$ that triggers emigration and all the result described above still apply. However, there is an

added selection in terms of the return decision. As we have show, individuals with high $\Omega(0)$, or low $S_1(0)$ return earlier, so those who remain in the receiving country become increasingly more likely to have a relatively high endowment of skill 1 which is more valued in the receiving country. Hence, the growth rate in the wages of the remaining immigrants exceed the growth rate implied by the increase in the rental rate and, on average, their local human capital rises faster than that of natives.