Review of Economic Studies (1986) LIII, 539-558
0034-6587/86/00360539$02.00

A Life-Cycle Consistent Empirical Model of Family Labour Supply Using Cross-Section Data

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This paper estimates a utility maximising model of the joint determination of male and female labour supplies using a sample of married couples from the U.K. Family Expenditure Survey. The emphasis is on the estimation of within period preferences that are consistent with intertemporal two-stage budgeting under uncertainty. However, the approach we adopt provides an alternative method of estimating certain aspects of life-cycle behaviour to the fixed effects λ-constant approach of Heckman and MaCurdy (1980), MaCurdy (1981) and Browning, Deaton and Irish (1985). Moreover, it relaxes some of the underlying restrictions that are implicit in these λ-constant models under uncertainty.

1. INTRODUCTION

Although the empirical study of labour supply has generally been cast in a static framework, a number of important results have recently been obtained within an explicitly intertemporal decision-making model (see, for example, Heckman (1974*b*), Ghez and Becker (1975), Smith (1977), Heckman and MaCurdy (1980), MaCurdy (1981, 1983), Browning, Deaton and Irish (1985) and Ham (1986)). However, underlying each of these studies are implicit restrictions on *within* period preferences, which are usually associated with the empirical approach adopted rather than with the life-cycle theory itself. For example, the empirical framework chosen by Heckman and MaCurdy (1980) and MaCurdy (1981) imposes within period additivity which clearly implies strong restrictions on behaviour (see Deaton (1974)). In view of the variety of labour supply behaviour observed in micro-data it is particularly important to allow a flexible representation of preferences as simple labour supply models often impose quite implausible restrictions on within period behaviour (see Stern (1986)).

Through the combination of intertemporal two stage budgeting and a dual representation of within period preferences, we generate a family labour supply model which is consistent with the life-cycle theory and nevertheless relaxes a number of the important underlying restrictions on within period preferences inherent in previous empirical models. Using a single cross-section of data we can only retrieve intertemporal elasticities with the addition of some identifying assumptions on intertemporal preferences (see MaCurdy (1983)), but these assumptions are common to a number of the models referred to above. Moreover, our estimates of within period preferences are invariant to such assumptions. In an intertemporal context the static specification, which has current labour supplies determined by the current marginal real wages and the current level of unearned income, is incorrect unless asset levels are planned or constrained to stay constant throughout the life-cycle. Current labour supply will depend not only on current assets and current real wages, but also on all future real wages. Thus the measurement of unearned income, which is a problem for the static model, does not arise directly in the intertemporal model. What is a problem, however, is the definition of a life-cycle consistent model in a form suitable for econometric estimation using available data. The source of this problem is that the whole life-cycle of real wages and their expectations is not available.

Ghez and Becker (1975) and Smith (1977) attempt to overcome the missing data problem by constructing synthetic cohorts using estimated life-cycle wage profiles from cross-section data on individuals of different ages. The synthetic cohorts are assumed to depict the life cycle of a representative individual. The major difficulty with this approach is that it tends to suffer from cohort bias and hence confounds cohort effects with true life-cycle effects. Many of the problems associated with the use of synthetic cohorts can be overcome by exploiting the repeated observations available in panel data, together with the important theoretical insights of Heckman (1974*a*), Heckman and MaCurdy (1980) and MaCurdy (1981). They show that under certain assumptions the individual's marginal utility of money (or λ) is, after suitable discounting, constant over the life-cycle, and that labour supply functions which condition on λ provide a suitable framework for both interpretation and estimation of life-cycle behaviour. The wage derivatives of such functions, for example, pick out exactly the responses to *anticipated* wage changes. Although unobservable, λ can be eliminated in estimation with panel data using the repeated observations available on each individual.

In all these models preferences are assumed intertemporally separable, so that individual decision-making can be viewed as a two-stage budgeting process, which makes the intuition behind the λ-constant approach clear. In the first stage the household allocates full life-cycle wealth across the life-time so as to equalise the marginal utility of (suitably discounted) money in all periods of the life-cycle. At the second stage, the current period's allocation of full income out of life-cycle wealth is distributed between consumption and non-market time depending on the level of the current real wages—the influence of all past and expected future variables is captured by the level of λ determined at the first stage.

This approach has been further developed by Browning, Deaton and Irish (1985) who consider the theoretical background to this λ-constant specification by relating it to the derivatives of the profit function representation of household preferences described in Gorman (1976). The attractions of their study are the generation of functional forms that break the within period additivity of the Heckman–MaCurdy specification and the use of pseudo panel data constructed from age cohort means across successive random cross-sections. This procedure avoids the cohort bias inherent in synthetic cohorts and has an advantage over true panel data to the extent that pseudo panels do not suffer attrition. However, averaging across cohorts inevitably reduces the underlying variation in dependent and explanatory variables as well as causing some difficulty in appropriately capturing the participation decision, especially important in the labour supply of married women which is a vital element of our family labour supply model.

As a vehicle for interpreting life-cycle labour supply behaviour the λ constant (or Frisch) framework is clearly very appealing. However in the estimation of life-cycle decisions under uncertainty or with replanning $\ln \lambda$ follows a random walk and is eliminated from the labour supply equation by first differencing. The way in which λ is allowed to enter the labour supply equation is consequently restricted. As a result not only are limitations imposed on the form of within period preferences, but also on the form of the "cardinalization" of utility. Indeed the estimates of within period preferences from the first differenced model will not be invariant to the form of the chosen cardinalization. Furthermore all current explanatory variables *not* fully anticipated become correlated with the stochastic error term and some instrumental variable procedure is required for estimation. Despite the relaxation of the within period additivity restriction of the Heckman–MaCurdy specification that is achieved in Browning, Deaton and Irish (1985) there remains some strong restrictions on within period preferences.

These points are developed in Section 2 along with an alternative approach which uses a flexible model imposing few restrictions on preferences and yet is consistent with the same underlying life-cycle optimizing model described above. The specification is generated from a dual representation of within period preferences allowing a general structure for wage and demographic variables. Household labour supplies that condition on the current period allocation out of life-cycle wealth, rather than on the marginal utility of money, are derived. Given the intertemporal separability assumptions this turns out to be a natural conditioning variable under two-stage budgeting and can easily be seen to capture all future anticipations and past decisions just as λ does in the λ-constant specification.

A similar approach is adopted by Altonji (1983) in his study of male labour supply where current food consumption is used to replace the unobservable marginal utility of money. Food expenditure then effectively summarizes all future anticipations. MaCurdy (1983), using an additive specification for within period preferences, exploits expenditure in the DIME panel data to estimate a male marginal rate of substitution function which is consistent with life-cycle planning. In our study we use the consumption expenditure data available in the U.K. Family Expenditure Survey.

In Section 3 the precise specification of our empirical application is described with particular attention to female participation, demographic variation and cohort effects. In Section 4 we consider the appropriate econometric estimation strategy and the results of applying it to a sample of married couples from the 1980 U.K. Family Expenditure Survey.

2. TWO-STAGE BUDGETING AND λ-CONSTANT FRISCH DEMANDS

A principal objective of this section is to compare the various alternative empirical parameterisations of the life-cycle decision making model and point to their implicit assumptions. In order to do so we employ a common optimising framework. Initially we shall assume that households have perfect foresight and choose current labour supplies and commodity demands so as to maximise discounted lifetime utility subject to budget, time and asset accumulation constraints. Following previous empirical studies in this area (see for example Heckman and MaCurdy (1980) and Browning, Deaton and Irish (1985)) we shall also assume intertemporal additive separability of life-cycle utility. Although additivity is not necessary for the application of two stage budgeting, it is invaluable when we come to relax the perfect foresight assumption. Separability of current from future decisions, on the other hand, is crucial throughout, enabling the influence of all past decisions over labour supply and commodity consumption to be summarised through the current level of assets.

Defining $\boldsymbol{x}^s$ to be the choice vector in period s, containing female and male non market time (l_{fs}, l_{ms}) and commodity consumption q_s, lifetime utility viewed from period t is written as the following discounted sum of concave and twice differentiable period by period utility indices $U_s(x^s)$,

$$V_t = \sum_{s=t}^{L} \delta^{s-t} U_s(\boldsymbol{x}^s) \tag{2.1}$$

where L is the lifetime horizon and δ represents a subjective time discount factor. The direct dependence of period by period utility on "s" reflects the influence of predetermined taste shifter variables, such as family size and composition variables, on life-cycle preferences.

Corresponding to $\boldsymbol{x}^s$ there is a price vector $\boldsymbol{p}^s$ containing female wage (w_{fs}), male wage (w_{ms}) and a commodity price index (p_{qs}) which define a within period budget identity

$$\boldsymbol{p}^s \boldsymbol{x}^s = y_s \tag{2.2}$$

where full income y_s has the form

$$y_s = w_{fs} T_f + w_{ms} T_m + \mu_s \tag{2.3}$$

with T_f and T_m as the total hours of market and nonmarket time available for women and men respectively. The variable μ_s is a measure of end of period s net dissaving and provides the crucial connection between current decisions and those in other periods. This point is easily seen by defining A_{s-1} to be the level of assets at end of period $s-1$ which earn interest at a rate r_s paid at end of period s and writing

$$\mu_s = r_s A_{s-1} - \Delta A_s \tag{2.4}$$

where ΔA_s is the additional asset allocation to period s. A negative value for ΔA_s, for example, reflects a movement of income out of period s and only for the case where $\Delta A_s = 0$ is the usual unearned income measure appropriate. This observation provides the important distinction between "static" and "dynamic" models under intertemporal separability. Clearly, where $\Delta A_s \neq 0$ the use of unearned income in the within period budget constraint is incorrect and can lead to a misinterpretation of within period behaviour (see MaCurdy (1982)).

The two constraints (2.2) and (2.4) are usefully combined into a lifetime wealth constraint

$$\sum_{s=t}^{L} \hat{\boldsymbol{p}}^s \cdot \boldsymbol{x}^s = W_t \tag{2.5}$$

where $\hat{\boldsymbol{p}}^s = \rho_s \boldsymbol{p}_s$ and ρ_s is the following market discount factor

$$\rho_s = \frac{1}{(1+r_s)(1+r_{s-1}) \cdots (1+r_{t+1})} \quad \text{for all } s > t \text{ and } \rho_s = 1 \text{ for } s = t.$$

Similarly total life-cycle wealth at end of period t, W_t is given by[1]

$$W_t = (1+r_t) A_{t-1} + \sum_{s=t}^{L} \hat{y}_s. \tag{2.6}$$

In addition to (2.5) there are inequality constraints on time: the upper bound of T_f on l_{fs} merits particular attention as a binding constraint in any period reflects female nonparticipation in market work.

The form of (2.1) and (2.5) is ideal for the application of two stage budgeting results (see Blackorby, Primont and Russell (1978), or Gorman (1968)) where at a first stage y_t is chosen so as to equalise the marginal utility of money in each period and at the second stage $\boldsymbol{x}^t$ is chosen *conditional* on y_t. These second stage demands can then be compared directly with the corresponding λ-constant Frisch demands of Heckman and MaCurdy (1980) and Browning, Deaton and Irish (1985) which condition on the marginal utility of money λ_t rather than on y_t. Although each is simply a reparameterisation of the same underlying optimising model described above, their corresponding empirical representations will be shown to differ in important ways especially when the perfect foresight assumption is relaxed.

Under two-stage budgeting the allocation of y_t is given by

$$y_t = \phi_t(\boldsymbol{p}^t, \hat{\boldsymbol{p}}^{t+1}, \ldots, \hat{\boldsymbol{p}}^L, W_t) \tag{2.7}$$

where ϕ_t is homogeneous of degree zero in discounted prices $\hat{p}^s$ and wealth W_t. It is clear from (2.7) that y_t summarizes the influence of all future economic and demographic variables on current period decisions as well as the influence of past decisions through A_{t-1} in W_t. The second stage allocation then determines within period demands according to

$$\boldsymbol{x}^t = \boldsymbol{g}^t(\boldsymbol{p}^t, y_t) \tag{2.8}$$

where $\boldsymbol{g}^t$ is a vector of demand equations homogeneous of degree zero in the price vector $\boldsymbol{p}^t$ and the conditioning variable y_t.

The forms of (2.7) and (2.8) are only correct if there are no binding constraints cn $\boldsymbol{x}^t$ in period t. Should, for example, the upper bound on female time bite, the form of $\boldsymbol{g}^t$ will generally change. This switch in preferences has been considered in Blundell and Walker (1983) and will be discussed further in Section 3 below. It is sufficient to note that the effect of this corner in the budget constraint is precisely the same as that observed in the standard static female participation model so long as the conditioning variables y_t is correctly measured irrespective of the regime. Binding constraints on $\boldsymbol{x}^s$ where $s > t$ will simply alter the form of (2.7) and will have no direct impact on (2.8). In each of these cases the measurement of y_t is crucial and where consumption data is available this can be achieved through the within period budget constraint without direct knowledge of the form of (2.7). Since the emphasis here is on estimating within period preferences and as we are able to measure y_t directly, the precise form that (2.7) takes across regimes is unimportant. Moreover, we shall see that under certain conditions intertemporal substitution elasticities can be recovered without knowing the future variables in (2.7).

As an alternative parameterization consider the familiar first order conditions (see Heckman and MaCurdy (1980)) for the maximization of (2.1) subject to (2.5)

$$\frac{\partial U_t}{\partial \boldsymbol{x}^t} = \lambda_t \boldsymbol{p}^t, \tag{2.9}$$

and

$$\lambda_{s+1} = \frac{1}{\delta(1+r_{s+1})}\lambda_s, \qquad s = t, \ldots, L. \tag{2.10}$$

The relationship (2.10) between the marginal utility of money in each period provides the link between the current and other period decisions analogous to (2.7). The variable λ_t acts as a summary of between period allocations and is a suitable conditioning variable for λ-constant demands which can be viewed as a rearrangement of (2.9) generating

$$\boldsymbol{x}^t = \boldsymbol{f}^t(\boldsymbol{p}^t, \lambda_t), \tag{2.11}$$

which are homogeneous of degree zero in $\boldsymbol{p}^t$ and λ_t.[2] The general properties of demand equations (2.11) are described in detail in Browning, Deaton and Irish (1985) and clearly provide a useful interpretation of life-cycle behaviour. For example, the wage elasticities identify the effect of (fully anticipated) movements along the household's lifetime wage profile. The drawback of working with (2.11) directly is that λ_t is unobservable and therefore information in (2.10) has to be exploited for empirical implementation. Once perfect foresight is relaxed this approach can generate implicit restrictions on the type of within period preferences that are permitted.

To illustrate these various points and to motivate our own empirical illustration consider representing within period preferences by the following general Gorman Polar Form indirect utility function

$$V_t = F_t\left[\frac{y_t - a_t(\boldsymbol{p}^t)}{b_t(\boldsymbol{p}^t)}\right] \tag{2.12}$$

where F_t is some concave transformation and the functions $a_t(\boldsymbol{p}^t)$ and $b_t(\boldsymbol{p}^t)$ are linear homogeneous functions of $\boldsymbol{p}^t$. Assuming there are no binding constraints the allocation of life-cycle wealth described by (2.7) may now be written as

$$y_t = a_t(\boldsymbol{p}^t) + \theta_t[b_t(\boldsymbol{p}^t), b_{t+1}(\hat{\boldsymbol{p}}^{t+1}), \ldots, b_L(\hat{\boldsymbol{p}}^L), W_t - \textstyle\sum_{s=t}^{L} a_s(\hat{\boldsymbol{p}}^s)] \tag{2.13}$$

where $a_s(\boldsymbol{p}^s)$ and $b_s(\boldsymbol{p}^s)$ act as price aggregators for across period allocations. From the application of Roy's identity to (2.12) within period demands will have the form

$$x_{it} = a_{it}(\boldsymbol{p}^t) + \frac{b_{it}(\boldsymbol{p}^t)}{b_t(\boldsymbol{p}^t)}[y_t - a_t(p^t)], \qquad i = f, m, q \tag{2.14}$$

where a_{it} and b_{it} are the derivatives of a_t and b_t. The particular forms of $a_t(p^t)$ and $b_t(p^t)$ will be given in the next section where emphasis is placed on their flexibility and the way in which they depend on demographic variables. However, it is worth pointing out that by choosing $a_t(p^t)$ to contain linear wage terms the T_f and T_m parameters can be subsumed into estimated parameters of the labour supply equations avoiding the need to choose them arbitrarily. While estimates of the parameters of (2.14) are independent of the choice of F_t in (2.12), in order to generate the corresponding λ-constant demands a particular cardinalization must be chosen. For example if F_t is chosen to be log linear and independent of t the marginal utility of income from (2.12) is simply

$$\lambda_t = \frac{1}{y_t - a_t(\boldsymbol{p}^t)}, \tag{2.15}$$

and the λ-constant demands corresponding to (2.14) take the form[3]

$$x_{it} = a_{it}(\boldsymbol{p}^t) + \frac{b_{it}(\boldsymbol{p}^t)}{b_t(\boldsymbol{p}^t)}\frac{1}{\lambda_t}. \tag{2.16}$$

Thus all λ-constant elasticities can be retrieved from the estimation of (2.14) given the choice of F_t. Clearly as λ_t is unobservable, (2.16) cannot be estimated directly but if panel data were available then λ_t could be written in terms of λ_0 from (2.10) which becomes a fixed effect for each household over the panel. Where panel data are not available but consumption data are, the alternative parameterisation (2.14) becomes invaluable. Indeed, when perfect foresight is relaxed the use of λ-constant demands for estimation can become quite restrictive.

The introduction of replanning or uncertainty leaves the y_t-conditional model almost unchanged since all uncertainty is captured through y_t in (2.14). For any choice of F_t the λ-constant wage elasticities, now reflecting fully anticipated movements along the wage profile, can be derived from (2.16). However, the evolution of λ_s in (2.10) is replaced by the following stochastic specification which ensures that λ_s is positive for all s,

$$\lambda_{s+1} = \frac{1}{\delta(1 + r_{s+1})}\lambda_s(1 + \varepsilon_{s+1}) \tag{2.17}$$

where ε_{s+1} reflects all unanticipated "news" gathered in period $s+1$ which, if exploited rationally, satisfies the condition[4]

$$E_s(\varepsilon_{s+1})=0. \tag{2.14}$$

Rewriting (2.17) as

$$\ln \lambda_{s+1}-\ln \lambda_s=\ln\left(\frac{1}{\delta(1+r_{s+1})}\right)+\xi_{s+1} \tag{2.19}$$

allows λ_s to be differenced out of λ-constant demands *provided* they are written linear in the logarithm of λ_t. Alternatively the left-hand side of (2.19) may be expressed as the marginal rate of intertemporal substitution as in Hansen and Singleton (1982). A general form for λ_t could be derived from (2.12) and estimation using (2.19) could proceed by generalising the approach of MaCurdy (1983). However, in order to generate the more popular linear differenced models restrictions have to be imposed on the form of F_t and the form of within period preferences.

Working with direct within period utility, the λ-constant linear differenced model requires that within period preferences be *explicitly* additive (see Heckman and MaCurdy (1980), MaCurdy (1981)). Browning, Deaton and Irish (1985), on the other hand, working with indirect preferences (2.12) generate a λ constant linear differenced model that allows within period additivity to be relaxed. However the price aggregator, $b_t(\boldsymbol{p}^t)$ is required to be Leontief restricting substitution possibilities for those with a high $y_t-a_t(\boldsymbol{p}^t)$, and once again the form of F_t is chosen prior to estimation. In these λ-constant models ξ_{t+1} becomes part of the disturbance term in the differenced demand equations so that all anticipated components of price, wage and demographic variables dated $t+1$ become correlated within the disturbances and suitable instruments for consistent estimation are required.

The alternative y_t-conditional parameterization adopted here overcomes a number of these problems since within period separability can be relaxed quite generally. Moreover, in our random effects models, specified in Section 4, there is no *theoretical* implication that any of the explanatory variables, in particular y_t, be correlated with the disturbances. Indeed, even though y_t is a choice variable, it may be reasonable to assume that disturbances on (2.8), (relating to choices within a period) are independent of those on (2.7) (relating to allocations across time), although clearly it would be desirable to test this assumption. The problem with developing such a test, as the discussion following (2.7) and (2.8) indicates, is that the form for y_t in (2.7) not only depends on future unobservable variables but also switches in the presence of binding current or future period constraints. From (2.15) it can be seen that the form of λ_t switches in a similar manner. Since the decision not to participate in the labour market can be represented by a binding constraint on available time, the form of y_t in (2.17) will exhibit an endogenous switching reduced form. Although some progress has been made in testing the independence assumption in the standard limited dependent variable model (see Smith and Blundell (1983)), the switching reduced form further complicates both testing this hypothesis and estimation under dependence.

3. AN EMPIRICAL SPECIFICATION OF FAMILY LABOUR SUPPLIES

For the empirical work presented here it is most convenient to work with the household's cost function which can be derived for our specification of preferences by inverting the indirect utility function (2.12). Dropping the time subscripts and ignoring for the moment the taste shifter variables, this is given by

$$C(w_f, w_m, p_q, U) = a(w_f, w_m, p_q) + b(w_f, w_m, p_q)G(U), \tag{3.1}$$

where $U = F^{-1}V$, $G(0) = 0$ and $C(\ ) = y$ at the optimum.

For interior solutions, where both partners choose positive hours of work the cost function is of the above form with the usual properties (see Deaton and Muellbauer (1980)). The wage derivatives of (3.1) yield Marshallian male and female labour supply equations on substituting the indirect utility function for U. However, should the female not participate in the current period then the household is at a corner solution with $l_f = T_f$ and the cost function takes on its "rationed" form. Following the literature on household decision making under rationing (see, for example, Latham (1980), Neary and Roberts (1980), and Deaton and Muellbauer (1981)) the relationship between the rationed and unrationed cost functions is given by

$$C^R(w_f, w_m, p_q, T_f, U) = C(w_f^*, w_m, p_q, U) + (w_f - w_f^*)T_f \tag{3.2}$$

where $C(\ )$ is (3.1) evaluated at the female reservation wage w_f^* at which the female would just choose to work. In Blundell and Walker (1983) we demonstrated that only when current female time is separable from other current choice variables is it the case that the functional form of the cost function does not change when a corner solution is attained. In the absence of within period separability, the cost function switches to its rationed form when nonparticipation occurs, even when the time path of w_f has been perfectly anticipated. Since the cost function switches at a corner solution, so too will the form of the male labour supply equation. That is, the derivatives of (3.2) with respect to w_m generates a male labour supply equation subject to the constraint that $h_f = 0$ which is different from that obtained from the derivative of (3.1) for $h_f > 0$. However, under both behavioural regimes the use of the conditioning variable y to produce labour supply equations that are consistent with life cycle optimisation remains legitimate. Thus the model is effectively one of switching regimes where female labour supply, itself an endogenous variable, acts as an indicator variable determining the sample separation.

A similar problem arises in Hausman and Ruud (1984) in the context of male and female labour supply with nonparticipation, and in Blundell and Walker (1982), in the context of labour supply and commodity expenditures. In Blundell and Walker (1982), rationing on male labour supply is analysed and the problem is simplified by using a preference specification that allows the virtual or reservation wage to be solved explicitly. In Hausman and Ruud (1984) rationing takes the form of female nonparticipation and the preference specification is such that the reservation wage, w_f^*, is defined implicitly by a quadratic equation which permits a relatively simple solution. Here rationing also occurs through nonparticipation but the preference specification is such that the reservation wage is defined by an implicit equation. However, our estimation technique is to select a sample of households for which $h_f > 0$ and use only the unrationed cost function to represent preferences. The resulting sample selection bias is then overcome through the appropriate sample likelihood outlined in Section 4. This avoids the need to estimate a matched pair of rationed and unrationed male labour supply equations and yet still identifies all the parameters of the model. It also has the advantage that it does not invoke in estimation the assumption that nonparticipation occurs purely because the (predicted) wage is less than the reservation wage. The form of the cost function (3.1) used here corresponds to a flexible version of Gorman Polar Form preferences (see Blackorby, Boyce and Russell (1980)) where quasi-homotheticity is imposed but not within period separability. The specification of $a(w_f, w_m, p_q)$, the cost of living at $U = 0$, is chosen to be second-order flexible but also allows for the possibility of fixed coefficients or zero substitution, and takes the generalised Leontief form

$$a(w_f, w_m, p) = \gamma_{ff} w_f + \gamma_{mm} w_m + \gamma_{qq} p_q + 2\gamma_{fm}(w_f w_m)^{1/2} + 2\gamma_{fq}(w_f p_q)^{1/2} + 2\gamma_{mq}(w_m p_q)^{1/2}.$$

In contrast, the form of $b(w_f, w_m, p_q)$ is chosen to be second order flexible nesting the substitution possibilities of the Cobb–Douglas case, and has the following Translog form

$$\ln b(w_f, w_m, p_q) = \beta_f \ln w_f + \beta_m \ln w_m + \beta_q \ln p_q + \frac{1}{2}\left[\beta_{ff} \ln\left(\frac{w_f}{p_q}\right)^2 + \beta_{mm} \ln\left(\frac{w_m}{p_q}\right)^2 + 2\beta_{fm} \ln\left(\frac{w_f}{p_q}\right) \cdot \ln\left(\frac{w_m}{p_q}\right)\right]$$

where $\beta_f + \beta_m + \beta_q = 1$. The γ and β coefficients are preference parameters, some or all of which may depend on taste shifter variables. The forms for $a(\ )$ and $b(\ )$ are such that the resulting y-conditional demand system nests the familiar Linear Expenditure System (by setting γ_{ij} for $i \neq j$ and all β_{ij} to zero), yet retains its convenient linearity in transformed variables. Written as labour supply equations, the system has the form

$$\begin{aligned} h_f &= \tilde{\gamma}_{ff} - \gamma_{fm}\left(\frac{w_m}{w_f}\right)^{1/2} - \gamma_{fq}\left(\frac{p_q}{w_f}\right)^{1/2} - \frac{\tilde{\beta}_f}{w_f}[\mu - \tilde{a}(w_f, w_m, p_q)], \\ h_m &= \tilde{\gamma}_{mm} - \gamma_{fm}\left(\frac{w_f}{w_m}\right)^{1/2} - \gamma_{mq}\left(\frac{p_q}{w_m}\right)^{1/2} - \frac{\tilde{\beta}_m}{w_m}[\mu - \tilde{a}(w_f, w_m, p_q)] \end{aligned} \tag{3.3}$$

where $\tilde{\gamma}_{ff} = T_f - \gamma_{ff}$, $\tilde{\gamma}_{mm} = T_m - \gamma_{mm}$, $\tilde{a}(w_f, w_m, p_q) = a(\ ) - w_f T_f - w_m T_m$, $\tilde{\beta}_f = \beta_f + \beta_{ff} \ln(w_f/p_q) + \beta_{fm} \ln(w_m/p_q)$, and $\tilde{\beta}_m = \beta_m + \beta_{fm} \ln(w_f/p_q) + \beta_{mm} \ln(w_m/p_q)$. Since μ is directly observable through the budget identity (3.3) can be estimated. Moreover estimation of (3.3) can proceed without making any assumptions about the values of T_f and T_m. Indeed if values for T_f and T_m are assumed, the estimated preference parameters will be independent of those assumptions.

Following the procedure in Section 2 the λ-constant labour supply functions corresponding to (2.16) are given by

$$\begin{aligned} h_f &= \tilde{\gamma}_{ff} - \gamma_{fm}\left(\frac{w_m}{w_f}\right)^{1/2} - \gamma_{fq}\left(\frac{p_q}{w_f}\right)^{1/2} - \frac{\tilde{\beta}_f}{w_f}\left(\frac{1}{\lambda}\right), \\ h_m &= \tilde{\gamma}_{mm} - \gamma_{fm}\left(\frac{w_f}{w_m}\right)^{1/2} - \gamma_{mq}\left(\frac{p_q}{w_m}\right)^{1/2} - \frac{\tilde{\beta}_m}{w_m}\left(\frac{1}{\lambda}\right). \end{aligned} \tag{3.4}$$

Under the particular choice of F in (2.12) the elasticities of the λ-constant functions are those with respect to evolutionary wage change over the life cycle and are of the form

$$\begin{aligned} E_{ff} &= \frac{1}{h_f}\left[\tilde{\gamma}_{ff} - \frac{1}{2}\gamma_{fm}\left(\frac{w_m}{w_f}\right)^{1/2} - \frac{1}{2}\gamma_{fq}\left(\frac{p_q}{w_f}\right)^{1/2} - \frac{\beta_{ff}}{w_f}\left(\frac{1}{\lambda}\right)\right] - 1, \\ E_{mm} &= \frac{1}{h_m}\left[\tilde{\gamma}_{mm} - \frac{1}{2}\gamma_{fm}\left(\frac{w_f}{w_m}\right)^{1/2} - \frac{1}{2}\gamma_{mq}\left(\frac{p_q}{w_m}\right)^{1/2} - \frac{\beta_{mm}}{w_m}\left(\frac{1}{\lambda}\right)\right] - 1. \end{aligned} \tag{3.5}$$

These correspond to male and female labour supply intertemporal substitution elasticities which theory dictates should be positive. The parameters required for the computation of (3.5) can be obtained from the estimation of (3.3), the Marshallian labour supply equations.

The deterministic specification of (3.3) is completed by allowing for the effects of taste shifter variables. The fact that our cross section consists of individuals of different ages suggests that some allowance should be made for different cohorts. That is, for instance, older women in the sample may have different leisure preferences simply because they were brought up at a time when female participation was less common than during the formative years of younger women in the sample. This suggests that β_f should be allowed to vary with female age, A. The role of dependent children is inevitably more complicated. The age of dependent children is likely to be extremely important as well as the number of children. Moreover it seems likely that there may be some economies of scale involved especially in their effect on the allocation of female time. With these aspects in mind, and with a view to economising on parameters while attempting to capture the wide variety of behaviour in the data we allow β_f to depend linearly on A, A^2, and dummy variables indicating the presence of the youngest child in each of three age groups; 0 to 4, 5 to 10, and 11 to 18. Thus we have

$$\beta_f=\beta_f^0+\beta_f' D'+\beta_f'' D''+\beta_f''' D'''+\beta_f^A(A-40)+\beta_f^{AA}(A-40)^2$$

where $D'=1$ if n', the number of children in the youngest group, is positive, $D''=1$ if $n''>0$ and $n'=0$, $D'''=1$ if $n'''>0$ and $n''=n'=0$. Such a specification ought to allow us to separate the cohort effect of female age from the effect of children on the marginal value of time. The role of the taste shifter variables in the γ parameters is given by

$$\begin{aligned}\gamma_{ff}&=\gamma_{ff}^0+\gamma_{ff}' n'+\gamma_{ff}'' D''+\gamma_{ff}''' D''',\\ \gamma_{mm}&=\gamma_{mm}^0+\gamma_{mm}' n'+\gamma_{mm}'' D''+\gamma_{mm}''' D''',\\ \gamma_{qq}&=\gamma_{qq}^0+\gamma_{qq}' n'+\gamma_{qq}'' n''+\gamma_{qq}''' n''',\end{aligned}$$

and

$$\gamma_{ij}=\begin{cases}\gamma_{ij}^0+\gamma_{ij}' n'+\gamma_{ij}''(n''D'')^{1/2}+\gamma_{ij}'''(n'''D''')^{1/2} & \text{for } i=m \text{ or } f, j=q\\ \gamma_{ij}^0+\gamma_{ij}' n'+\gamma_{ij}'' D''+\gamma_{ij}''' D''' & \text{for } i=f, j=m.\end{cases}$$

Such a specification allows for the possibility of economies of scale on male and female time but not directly on consumption. Further details of this demographic specification can be found in Blundell and Walker (1984).

4. ESTIMATION, DATA AND RESULTS

4.1. *A Maximum Likelihood Estimator*

For estimation purposes, a sample of families with working wives were selected and additive disturbances (random effects) assumed for the earnings and expenditure equations derived in Section 3. As these disturbances will very likely be heteroskedastic with a variance related to the overall level of income in the household, each equation was deflated by full income so that the dependent variables become budget shares. As is usual in demand analysis the system of estimating equations is overdetermined and all information on preferences can be recovered after the arbitrary deletion of one equation. However, the selection rule for households invalidates the use of seemingly unrelated estimation procedures since such estimators would suffer from selectivity bias as described in Heckman (1979). Indeed, the adding-up condition implies that selectivity bias will affect equations other than the female time equation over which the selection takes place (see Blundell and Walker (1983)).

Estimation methods that correctly account for selectivity fall into two categories. The first are the two-stage methods of Heckman (1979), Lee, Madalla and Trost (1980) and Hanoch (1980). These are inefficient in comparison with full maximum likelihood procedures and require reasonably complicated algebraic manipulation to derive asymptotic covariances. Given that our system contains only two equations (after deletion), full maximum likelihood estimation which comprises the second group of estimators is feasible. The form of the likelihood for truncated samples is described in Hausman and Wise (1977), and Wales and Woodland (1980). It is a mixture of density and distribution functions with similar properties to the Tobit likelihood described in Tobin (1958) and Amemiya (1973).

If we let $\phi(u_{fh}, u_{mh})$ represent the joint density of the disturbances on the female time and male time equations in share form, then the likelihood for a sample of H households is given by:

$$\prod_{h=1}^{H} \phi(u_{fh}, u_{mh})/\Pr(l_{fh} < T_f) \tag{4.1}$$

where $\Pr(l_{fh} < T)$ is the probability that household h is selected. To maximise the log of (4.1) a nonlinear iterative technique is required: the method adopted here is a version of the quasi-Newton algorithm, E04JBF from the NAG Fortran Library, described in Gill and Murray (1972). As the truncated likelihood (4.1) cannot be made globally concave, particular care was taken to ensure that a global maximum was attained in all cases.

4.2. *Data*

In order to implement the specification outlined in Section 3 we require a source of data that contains hours worked, wages and either consumption expenditure directly or saving so as to construct consumption expenditure via the budget and asset accumulation constraints. Such information is available from the U.K. Family Expenditure Surveys and here we use a subset from the 1980 FES. The subset was chosen so that all households consisted of two married working employees (FES code A201 = 1) with the head of household either a manual worker (code A210 = 6, 7 and 8), a shop assistant (A210 = 5) or a clerical worker (A210 = 4). The resulting sample contained 1378 households giving a female participation rate of around 64%. The hourly gross wages, assumed exogenous, for both husband and wife were constructed as the gross earnings to hours ratio where the "normal" definitions of these variables were used in an attempt to minimise the measurement error problem.[5] The method used to calculate the households' marginal (after tax) hourly wage rates is briefly described in the Data Appendix along with summary statistics for each variable in the sample.

4.3. *Empirical Results*

The parameter estimates, their standard errors and corresponding log likelihood values for two models of particular interest are presented in Tables I(a) and I(b). Model 1(a) refers to the preferred specification chosen by the usual asymptotic likelihood ratio criterion from among those that adopt a Cobb–Douglas form for the price index $b(w_f, w_m, p)$ while Model 1(b) is the preferred specification from among those that adopt a Translog form. From the log likelihood values it is clear that Model 1(b) is the overall preferred specification. However, it is clear from both specifications that certain behavioural features of the household allocation of time and goods are robust to the change in specification.

Turning first to the β coefficients which determine the marginal values of time and goods we found that the β_m coefficients are not significantly sensitive to composition and

TABLE I(a)

Model estimates with $b(\mathbf{p})$ Cobb–Douglas

	Female time		Male time		Goods expenditure	
β_i^0	0·122	(0·021)	0·089	(0·008)	0·789	(0·021)
β_i'	0·191	(0·039)	0		−0·191	(0·039)
β_i''	0·160	(0·037)	0		−0·160	(0·037)
β_i'''	0		0		0	
β_i^A	0·0067	(0·001)	0		−0·0067	(0·001)
β_i^{AA}	0·0001	(0·0001)	0		−0·0001	(0·0001)
β_{fi}	0		0		0	
	61·52	(1·472)	0		−8·48	(1·894)
γ_{ij}^0			49·47	(0·249)	0	
					50·64	(2·489)
	9·70	(1·400)	0		8·25	(1·964)
γ_{ij}'			0		0	
					7·77	(2·974)
	4·59	(2·480)	0		8·15	(2·747)
γ_{ij}''			0		0	
					1·62	(1·783)
	8·73	(1·701)	0		0·72	(1·663)
γ_{ij}'''			0		0	
					11·84	(1·546)
	0·0028	(0·0001)	−0·0003	(0·00006)	−0·0025	(0·0001)
σ_{ij}			0·0015	(0·00006)	−0·0012	(0·00009)
					0·0037	(0·0001)

Note: The data for explanatory and dependent variables are summarized in the data appendix. Exclusion restrictions are indicated by 0. Figures in parentheses are asymptotic standard errors. $\ln L = 4815{\cdot}0$.

TABLE I(b)

Model estimates with $b(\mathbf{p})$ translog

	Female time		Male time		Goods expenditure	
β_i^0	0·144	(0·022)	0·089	(0·009)	0·767	(0·022)
β_i'	0·198	(0·039)	0		−0·198	(0·039)
β_i''	0·162	(0·038)	0		−0·162	(0·038)
β_i'''	0		0		0	
β_i^A	0·0064	(0·001)	0		−0·0064	(0·001)
β_i^{AA}	0·001	(0·0001)	0		−0·0001	(0·0001)
β_{fi}	0·249	(0·031)	0		−0·249	(0·031)
	52·03	(2·477)	0		2·33	(2·641)
γ_{ij}^0			49·47	(0·247)	0	
					41·25	(5·092)
	13·10	(3·519)	0		2·88	(4·296)
γ_{ij}'			0		0	
					12·02	(6·540)
	6·88	(2·630)	0		3·67	(2·542)
γ_{ij}''			0		0	
					0·52	(2·445)
	7·62	(2·645)	0		0·93	(2·383)
γ_{ij}'''			0		0	
					11·90	(2·319)
	0·0026	(0·0001)	−0·0003	(0·00006)	−0·0023	(0·0001)
σ_{ij}			0·0015	(0·00006)	−0·0012	(0·00009)
					0·0035	(0·0002)

Note: See Table I(a). $\ln L = 4844{\cdot}9$.

cohort changes and are therefore restricted to zero in these preferred specifications. Thus, the strong positive effects of the presence of pre-school and junior school children in the household on the marginal value of female time is reflected in the marginal value of goods expenditure. Similarly, the strong, almost linear, positive effect of female age on the marginal value of female time is reflected in its negative effect on the marginal value of goods. Thus, a 60-year old female will have a marginal value of time of approximately 0.26 in excess of that for an otherwise identical 20-year old; a difference which we interpret as a reflection of the change in tastes across cohorts but could also be attributed to vintage effects. Strong cohort effects on female labour supply are no surprise and have been reported elsewhere; for example in Greenhalgh (1977). In Table I(b) the index $b(w_f, w_m, p_q)$ takes the Translog form and the coefficient of 0·249 is that on $\ln(w_f/\bar{w}_f)$ so as to maintain the comparability of the β coefficients. This coefficient implies a large wage impact on the marginal value of time; for example, a wage of 10% above the mean would add almost 0·1 to the marginal value of time. The most important differences between the two specifications lies in the contrasting ways in which the female real wage enters the equations.′ In Table I(b) with the Translog specification, w_f enters via both the $a(w_f, w_m, p_q)$ and $b(w_f, w_m, p_q)$ functions, while in Table I(a) w_f enters only via the $a(w_f, w_m, p_q)$ function. Both specifications imply that preferences between goods and female time are nonseparable. Notice that the inclusion of $\ln w_f$ in the marginal budget shares of Table I(b) appears to be strongly significant while the γ^0_{fq} parameter is small and insignificant. Thus, the Translog specification rejects separability through the significance of $(\ln w_f/\bar{w})^2$ in $b(w_f, w_m, p_q)$ while the Cobb–Douglas specification rejects separability through the significance of w_f in $\partial a(w_f, w_m, p_q)/\partial p_q$. Since the inclusion of $\ln w_f$ in the consumption function turns γ^0_{fq} from being significantly negative to insignificantly different from zero the diagonal elements γ^0_{ff} and γ^0_{qq} are correspondingly smaller.

The specification of the male equation is, in both tables, such that male time is additively separable from goods and female time so that this is essentially a Linear Expenditure System form. Thus, the interpretation of γ^0_{mm} is the usual one of subsistence leisure time. Since there is little variation in male labour supply to be explained such a simple specification does not seem inappropriate.

Analogous to the Linear Expenditure System we can evaluate subsistence quantities. Each row or column of the γ^0 matrix can, after suitable weighting, be summed to give the predicted expenditure on female time, male time and goods evaluated at $U=0$ for households with no children. Thus, using Table I(a) and mean wages, a two adult household subsistence female time is 54·2 hours ($T_f=90$), subsistence male time is 49·5 hours ($T_m=90$) and subsistence goods expenditure is £40·80.

Male labour supply is restricted to be independent of the presence and number of children except to the extent that they affect supernumary income, $y-a(w_f, w_m, p_q)$. The effect of children on subsistence costs is allowed through the γ', γ'', and γ''' matrices which show that the number of children have a large effect on 'necessary' expenditure and that the presence of children has a large effect on 'necessary' female time. These have an unambiguously positive effect on male hours of work indirectly through increasing $a(w_f, w_m, p_q)$. However, the effects of children are more complicated in the consumption and female time functions. The positive effect of the increase in subsistence goods expenditure on female labour supply tends to be offset by negative effects of the increase in subsistence female time. The net effect is likely to be negative for a young child and positive for an old child, and the negative effect of a young child is reinforced by its large effect on the marginal value of female time.

Finally, Tables I(a) and I(b) include the estimated variance-covariance matrices. Both sets of covariances indicate that given the underlying distributional assumption the selection bias in the female equation significantly affects the male and goods equations.

Although these models were selected on the basis of the standard likelihood ratio criterion after appropriately adjusting for selectivity, the statistical properties of our estimated parameters depend critically on the appropriateness of the stochastic assumptions in our random effects specification. In particular, the joint normality of the error distribution across the equation system and the independence of explanatory variables with these errors. Thus an important area for future research is the development of reliable diagnostics for these assumptions in systems of this type.

Turning to the underlying properties of the models it is clear that the interactions in the models are complicated and since the elasticities are sensitive to the points at which they are evaluated we have selected a number of subsamples from the full sample (the means of the subsamples data are presented in Table II(a)). The subsamples were selected in an attempt to represent a typical life cycle by selecting successively older women with a time path of fertility involving two children born five years apart, the first in her mid-twenties. The children grow older and eventually leave the household when the mother is in her early fifties. Since the households conforming to the snapshots of such a life cycle profile were selected from the single cross section the subsamples inevitably confound life cycle and cohort effects; and since many households do not conform to such a profile the subsamples do not exhaust the complete sample used in estimation. Nevertheless, this procedure is useful to check the predictive performance of the estimated equations and to examine the sensitivity of elasticities. In Table II(b) the predicted hours of males and females are both corrected for the selectivity bias in the female equation the predicted consumption figures are derived from the budget constraint. All predictions were obtained by calculating them for each relevant data point and taking the mean, rather than calculating them for the mean of the relevant subsamples. In general both sets of estimates tend to underpredict hours and hence underpredict consumption also. For the mean of the whole sample the error is greatest for female hours, but this is not surprising given the larger variation compared with male hours and consumption. Since there are relatively large changes in μ and w_m as well as female age across the subsamples it is difficult to conclude from the relative sizes of the errors that the model is misspecified in any particular systematic way, and the fact that our predictions are at most 8% low is reassuring. The male equation does not seem to track the variation in actual hours particularly well but this is not too important given the limited variation in the data. The consumption equation seems to capture the subsample data reasonably well except that it doesn't entirely capture the large increases for groups 4 and 5. Table II(b) also presents labour supply and expenditure elasticities for the various samples using both specifications. The elasticities were calculated by computing the elasticities at each relevant data point and taking means. The full income elasticities confirm previous work and show that male time is a normal good, and female time more strongly so except where the female is young and without children, when the two elasticities take broadly similar magnitudes. The compensated own price/wage elasticities take the appropriate sign when averaged over the whole sample but occasionally contradict economic theory when averaged over the subsamples. The compensated cross elasticities are negative except for e_{fq} which change sign across subsamples for both sets of parameter estimates. Thus male and female time are complements while female time and consumption as one would expect are often substitutes. Such a pattern of substitution could not arise for normal goods if separability were a maintained hypothesis. Through the Slutsky equation we can retrieve

TABLE II(a)

Sample statistics for selected household types

	Age range	Mean age	Household composition						Sample size	Marginal female wage (£)	Marginal male wage (£)	Female hours	Male hours	Net dissaving (£)	Consumption (£)
		$\bar{A}$	n'	n''	n'''	D'	D''	D'''		w_f	w_m	h_f	h_m	μ	q
1	16–30	24·2	0	0	0	0	0	0	251	1·39	1·90	36·1	39·2	−59·89	64·67
2	20–35	26·9	1	0	0	1	0	0	48	1·46	1·88	20·1	39·3	−34·22	69·98
3	25–40	31·0	1	1	0	1	0	0	69	1·40	2·08	17·9	40·2	−35·14	73·65
4	30–45	37·1	0	1	1	0	1	0	85	1·30	2·31	23·0	40·0	−38·83	83·34
5	35–50	42·0	0	0	1	0	0	1	78	1·35	2·14	24·4	40·9	−34·78	85·72
6	40–60	52·1	0	0	0	0	0	0	268	1·35	1·99	29·5	39·6	−48·72	69·75
Total sample	16–60	37·1	0·182	0·463	0·458	0·152	0·253	0·169	1378	1·38	2·08	26·6	39·8	−45·30	74·37

TABLE II(b)

Mean predictions and elasticities for selected household types

		Selectivity adjusted predictions			λ constant elasticities			Compensated elasticities						Income elasticities					
		$\hat{h}_f$	$\hat{h}_m$	$\hat{q}$	E_{ff}	E_{mm}	E_{qq}	e_{ff}	e_{fm}	e_{fq}	e_{mm}	e_{mq}	e_{qq}	η_f	η_m	η_q	$y-\hat{a}$	$\hat{\beta}_f$	$\hat{\beta}_q$
Model (a)	1	35·3	39·4	61·04	−0·089	0·029	−0·110	−0·080	−0·002	0·083	0·028	−0·026	0·022	−0·206	−0·280	3·281	26·01	0·04	0·87
Cobb–Douglas	2	18·5	40·0	67·99	0·084	0·010	−0·098	0·083	−0·007	−0·059	0·013	−0·008	−0·047	−2·542	−0·314	2·746	11·59	0·24	0·67
model	3	18·1	39·7	72·78	0·143	0·013	−0·134	0·135	−0·013	0·103	0·016	−0·010	−0·057	−3·13	−0·297	2·607	15·99	0·26	0·65
	4	21·6	39·5	80·50	0·191	0·020	−0·171	0·165	−0·019	−0·132	0·020	−0·013	−0·068	−2·867	−0·277	2·387	22·33	0·26	0·65
	5	24·2	39·1	81·56	−0·012	0·033	−0·152	−0·013	−0·012	0·033	0·033	−0·027	−0·028	−1·203	−0·295	2·743	34·20	0·14	0·78
	6	30·1	38·7	68·93	0·080	0·046	−0·175	0·046	−0·019	−0·021	0·041	−0·031	−0·058	−1·367	−0·290	2·588	39·19	0·22	0·69
Whole sample		25·9	39·5	72·60	0·028	0·024	−0·137	0·032	−0·010	−0·014	0·024	−0·018	−0·038	−1·323	−0·287	2·800	23·81	0·17	0·74
Model (b)	1	34·8	39·5	63·53	−0·055	0·027	−0·457	−0·057	−0·003	0·061	0·025	−0·022	0·013	−0·369	−0·279	3·141	31·13	0·08	0·84
Translog model	2	18·8	39·9	68·24	0·129	0·013	−0·292	0·102	−0·011	−0·077	0·014	−0·008	−0·053	−2·930	−0·315	2·481	12·51	0·29	0·62
	3	17·9	39·6	72·29	0·162	0·019	−0·344	0·102	−0·019	−0·075	0·018	−0·011	−0·045	−3·519	−0·298	2·423	17·92	0·30	0·61
	4	21·1	39·3	79·38	0·150	0·026	−0·441	0·079	−0·025	−0·045	0·025	−0·017	−0·043	−2·977	−0·278	2·354	27·72	0·27	0·64
	5	24·3	39·2	81·91	−0·042	0·033	−0·463	−0·061	−0·013	0·079	0·031	−0·025	−0·005	−1·305	−0·295	2·637	32·26	0·15	0·76
	6	29·8	38·9	68·92	0·018	0·041	−0·546	−0·029	−0·019	0·052	0·038	−0·028	−0·009	−1·466	−0·288	2·506	36·29	0·23	0·68
Whole sample		25·6	39·5	72·19	0·033	0·026	−0·433	0·009	−0·012	0·007	0·024	−0·018	−0·026	−1·495	−0·287	2·669	24·81	0·20	0·71

Note: All elasticities are for labour supplies or expenditure. Income elasticities are full income elasticities. Predictions $\hat{h}_f$, $\hat{h}_m$ and $\hat{q}$ are adjusted for the selection rule $h_f > 0$.

the uncompensated elasticities which hold μ_t constant. The female labour supply elasticity is generally positive for both specifications.

Given a particular choice of the cardinalization, F, in the indirect utility function (2.12) we can recover λ-constant intertemporal substitution elasticities from the estimated y-conditional equations. For example, choosing F to be log linear generates the elasticities in Table II(b). Under the assumptions of a perfect credit market and rational expectations the λ-constant labour supply elasticities are required to be positive and the λ-constant consumption elasticity is required to be negative. Unlike intertemporal elasticities derived in the λ-constant approach of Heckman and MaCurdy (1980) and MaCurdy (1981) our specification allows intertemporal substitution effects to be data dependent. The variations in the data across the subsamples results in quite large variations in the elasticities. Again, the female own intertemporal elasticity violates economic theory for some subsamples, but otherwise the elasticities take the appropriate sign.

Finally, in Table III we provide the comparative statics of a large change in net dissaving on the behaviour of six hypothetical households. One interpretation of the table is the effects of a decrease in unearned income of £50 in the static model—that is, the model with perfect credit rationing. In this case the behavioural changes given by the predicted hours, consumption, and female participation $\hat{\Phi}_f$ are the result of pure income effects. The life-cycle model offers two alternative interpretations. Low net saving in Table III(a) may arise from either high initial assets or in anticipation of high future wage growth. Thus, under the life-cycle interpretation the households in III(a) are currently identical to the corresponding households in III(b) but differ in respect to either their pasts or their anticipated futures. In the first case the behavioural responses are the

TABLE III

Model simulations for hypothetical households

(a) $w_f=£1{\cdot}00$, $w_m=£2{\cdot}00$, $\mu=-£10{\cdot}00$

	Composition			Selectivity adjusted predictions						
Age	n'	n''	n'''	$\hat{h}_f$	$\hat{h}_m$	$\hat{q}$	$\hat{\beta}_f$	$\hat{\beta}_q$	$y-\hat{a}$	$\hat{\Phi}_f$
25	0	0	0	34·2	37·6	99·17	0·05	0·87	65·85	0·99
25	1	0	0	16·3	38·3	79·70	0·24	0·68	31·88	0·80
30	1	1	0	16·2	38·4	79·63	0·26	0·66	30·26	0·79
35	0	1	1	19·4	38·6	84·58	0·25	0·66	31·49	0·88
40	0	0	1	23·7	38·3	89·33	0·12	0·79	43·83	0·95
50	0	0	0	25·1	37·4	89·04	0·20	0·71	65·85	0·96

(b) $w_f=£1{\cdot}00$, $w_m=£2{\cdot}00$, $\mu=-£60{\cdot}00$

	Composition			Selectivity adjusted predictions						
Age	n'	n''	n'''	$\hat{h}_f$	$\hat{h}_m$	$\hat{q}$	$\hat{\beta}_f$	$\hat{\beta}_q$	$y-\hat{a}$	$\hat{\Phi}_f$
25	0	0	0	36·3	39·8	55·88	0·05	0·87	15·85	1·00
25	1	0	0	23·8	41·3	45·95	0·24	0·68	−18·12	0·98
30	1	1	0	24·5	41·3	46·87	0·26	0·66	−19·74	0·99
35	0	1	1	29·0	41·3	51·55	0·25	0·66	−18·51	0·99
40	0	0	1	28·4	40·8	49·86	0·12	0·79	−6·17	0·99
50	0	0	0	33·8	39·8	53·45	0·20	0·71	15·86	1·00

result of pure property income effects. In the second case, where the households in III(a) anticipate high future wages relative to the current wage and those in III(b) anticipate low future wages, the behavioural effects are the result of both intertemporal substitution and wealth effects. Under additive intertemporal separability such substitution effects are proportional to income effects.

5. SUMMARY AND CONCLUSION

This paper presents estimates of a model of family labour supply which allows for quite general effects of relative wages and demographic variables on within period behaviour and yet is consistent with life-cycle optimising behaviour under intertemporal separability and uncertainty. Furthermore, providing some identifying assumptions are made on the cardinalization of utility, certain useful intertemporal substitution elasticities can be retrieved. These identifying assumptions turn out to be no more restrictive than those imposed in many of the popular λ-constant life-cycle labour supply models. Moreover, our model relaxes a number of the underlying restrictions on within period preferences implicit in the empirical formulation of these alternative specifications and our estimates of within period preference parameters are invariant to the chosen cardinalization. Indeed, one of the motivations for this study was to highlight the importance of allowing for general substitution and demographic effects in the within period allocation of time and goods. These effects were found to be critically important in the analysis of family labour supply presented here.

For our empirical application we chose a large sample of working couples from the 1980 U.K. Family Expenditure Survey. The advantage of this survey is that it not only has reliable earnings and hours data but also collects information on commodity expenditures which, following the important work of MaCurdy (1982, 1983), allow the unobservable marginal utility of wealth in the λ-constant formulation to be replaced by an observable full-income variable. It is this replacement that permits the relaxation of the implicit restrictions on within period preferences underlying the empirical formulations of the λ-constant model. The estimated models confirm the need for a flexible representation of preferences over time and goods. As one would expect, the hours of work of women in the sample appear to be much more responsive to changes in marginal wage and income variables than is the case for men. Simple models such as the Linear Expenditure System, which implies linear earnings equations, are easily rejected by the data as are models that restrict the interaction of demographic and economic variables. The resulting estimates for our preferred models were found to track the large variety of observed behaviour across different subsamples relatively well, and the estimated elasticities were generally found to be consistent with economic theory.

DATA APPENDIX

Gross wages were calculated from the ratio of normal earnings (A080) to normal hours (A220). The amount of taxable income was deduced from: the date of interview (since tax allowances varied across the year), interest payments on mortgages (code 130 or 150), life insurance premium (codes 196 and 199) and superannuation (pension) contributions (code 318). National insurance contributions were calculated from the date of interview and individual earnings. Entitlement to Family Income Supplement was calculated from the date of interview, the number of children and gross household income excluding child benefit. The appropriate marginal deduction rate was then applied to the gross hourly wage as described in Blundell, Meghir, Symons and Walker (1986). On the accuracy of the FES data on earnings and hours see Atkinson, Micklewright and Stern (1982*a*, *b*). On the overall degree of nonresponse, see Kemsley (1977). Total household expenditure is defined as expenditure on energy, food, clothing and footwear, services and other goods (the sum of codes 368P, 369P, 376P, 372P and 374P), which excludes housing, alcohol, tobacco, durables, transport and vehicles, and miscellaneous. Finally, since the data are drawn from households surveyed throughout the year, expenditure and the marginal wages were adjusted for inflation over the year using a price index constructed from the relevant components of the retail price index.

		Mean	Standard deviation
Female leisure share	s_f	0·328	0·102
Male leisure share	s_m	0·397	0·098
Female hours of work per week	h_f	26·64	11·575
Male hours of work per week	h_m	39·79	4·771
Female net marginal wage (£)	w_f	1·384	0·791
Male net marginal wage (£)	w_m	2·081	0·853
Energy expenditure (£/week)		6·51	6·44
Food expenditure (£/week)		29·55	11·22
Clothing and footwear expenditure (£/week)		11·73	14·25
Services expenditure (£/week)		14·58	32·10
Other goods expenditure (£/week)		10·61	10·15
Number of children, 0–4	n'	0·182	0·461
Number of children, 5–10	n''	0·463	0·737
Number of children, 11–18	n'''	0·458	0·779
Female age	A	37·11	10·48

First version received March 1984; *final version accepted September* 1985 (*Eds.*)

Many of the ideas in this paper were developed while the authors were attending the ESRC Summer Workshop on the Microeconomic Analysis of Labour Markets held at Warwick in July 1982. We are indebted to the participants at that workshop and those in subsequent seminars at Bristol, Birmingham, LSE, Manchester, Nuffield and Princeton for their comments. We are particularly indebted to Martin Browning, John Ham, Grayham Mizon, Costas Meghir, Mark Stewart and two referees for their comments. Funding for this research was provided by ESRC grants D0023 0004 and B2005 2060 and the data was supplied by the ESRC Data Archive. Tim Barmby and Elizabeth Symons provided excellent research assistance.

NOTES

1. This formulation of the life-cycle problem assumes $A_L = 0$ and abstracts from any bequest motive. However, the empirical formulation below is also consistent with an intertemporally separable bequest argument in the utility function but is unable to identify its parameters.
2. The functional form of (2.11) remains the same at a corner solution. However, the relevant price would be replaced by a virtual price, see Browning, Deaton and Irish (1985).
3. Such a form is a generalisation of the life-cycle model employed by Ashenfelter and Ham (1979).
4. In the context of a life-cycle consumption see also Hall (1978) and Muellbauer (1983).
5. In a linear model it is possible, under certain circumstances (see Borjas (1980)), to use the actual hourly wage as an instrument for the normal hourly wage providing a consistent estimator under measurement error. For our nonlinear selectivity adjusted model this may also be a useful approach but the properties of such an instrumental variable estimator would clearly need some further investigation.

REFERENCES

ALTONJI, J. G. (1983), "Intertemporal Substitution in Labour Supply: Evidence from Micro-Data", *Journal of Political Economy* (forthcoming).

AMEMIYA, T. (1973), "Regression Analysis when the Dependent Variable is Truncated Normal", *Econometrica*, **41**, 997-1017.

ASHENFELTER, O. and HAM, J. (1979), "Education, Unemployment and Earnings", *Journal of Political Economy*, **87**, S99-S166.

ATKINSON, A. B., MICKLEWRIGHT, J. and STERN, N. H. (1982), "A Comparison of the Family Expenditure Survey and the New Earnings Survey 1971-1977: Part I: Characteristics of the Sample" (Discussion Paper 27, SSRC Programme on Taxation, Incentives and the Distribution of Income, LSE).

ATKINSON, A. B., MICKLEWRIGHT, J. and STERN, N. H. (1982*b*), "A Comparison of the Family Expenditure Survey and the New Earnings Survey 1971-1977: Part II: Hours and Earnings" (Discussion Paper 32, SSRC Programme on Taxation, Incentives and the Distribution of Income, LSE).

BLACKORBY, C., PRIMONT, D. and RUSSELL, R. R. (1978) *Duality, Separability and Functional Structure* (New York: North-Holland).

BLUNDELL, R. W., MEGHIR, C., SYMONS, E. and WALKER, I. (1986), 'A Labour Supply Model for the Simulation of Tax and Benefit Reforms', in Blundell, R. W. and Walker, I. (eds.), *Unemployment, Search and Labour Supply* (Cambridge: Cambridge University Press).

BLUNDELL, R. W. and WALKER, I. (1982), "Modelling the Joint Determination of Household Labour Supplies and Commodity Demands", *Economic Journal*, **92**, 351-364.

BLUNDELL, R. W. and WALKER, I. (1983), "Limited Dependent Variables in Demand Analysis: An Application to Modelling Family Labour Supply and Commodity Demand Behaviour", (Discussion Paper in Econometrics ES 126, University of Manchester).

BLUNDELL, R. W. and WALKER, I. (1984), "A Household Production Specification of Demographic Variables in Demand Analysis", *Economic Journal*, **94**, 59-67.

BORJAS, G. J. (1980), "The Relationship Between Wages and Weekly Hours of Work: The Role of Division Bias", *Journal of Human Resources*, **15**, 409-421.

BROWNING, M. J., DEATON, A. S. and IRISH, M. (1985), "A Profitable Approach to Labour Supply and Commodity Demands over the Life-Cycle", *Econometrica*, **53**, 503-544.

DEATON, A. S. (1974), "Reconsideration of the Empirical Implications of Additive Preferences", *Economic Journal*, **84**, 338-348.

DEATON, A. S. and MUELLBAUER, J. (1981), "Functional Forms for Labour Supply and Commodity Demands with and without Quantity Restriction", *Econometrica*, **49**, 1521-1532.

GHEZ, G. and BECKER, G. S. (1975) *The Allocation of Time and Goods Over the Life Cycle* (New York: Columbia University Press).

GILL, P. E. and MURRAY, W., "Quasi-Newton Methods for Unconstrained Optimisation", *Journal of the Institute of Mathematics and its Applications*, **9**, 91-108.

GORMAN, W. M. (1968), Conditions for Additive Separability', *Econometrica*, **36**, 606-609.

GORMAN, W. M. (1976), "Tricks with Utility Functions", in M. J. Artis and A. R. Nobay (eds.), *Essays in Economic Analysis* (Cambridge: Cambridge University Press).

GREENHALGH, C. (1977), 'A Labour Supply Function for Married Women in Great Britain', *Economica*, **44**, 249-265.

HALL, R. E. (1978), "Stochastic Implications of the Life-Cycle Permanent Income Hypothesis: Theory and Evidence", *Journal of Political Economy*, **86**, 971-988.

HAM, J. C. (1986), 'Testing Whether Unemployment Represents Intertemporal Labour Supply Behaviour', *Review of Economic Studies*, (this issue).

HANOCH, G. (1980), "A Multivariate Model of Labour Supply: Methodology and Estimation", in J. P. Smith (ed.), *Female Labour Supply: Theory and Estimation* (Princeton: Princeton University Press).

HANSEN, L. P. and SINGLETON, K. J. (1982), "Generalised Instrumental Variables Estimation of Non-Linear Rational Expectations Models", *Econometrica*, **50**, 1269-1286.

HAUSMAN, J. A. and RUUD, P. (1984), "Family Labour Supply with Taxes", *American Economic Review (Papers and Proceedings)*, **74**, 242-248.

HAUSMAN, J. A. and WISE, D. A. (1977), "Social Experimentation, Truncated Distribution and Efficient Estimation", *Econometrica*, **45**, 913-938.

HECKMAN, J. J. (1974*a*), "Life Cycle Consumption and Labour Supply: An Explanation of the Relationship Between Income and Consumption Over the Life Cycle", *American Economic Review*, **64**, 188-194.

HECKMAN, J. J. (1974*b*), "The Effect of Child Care Programs on Women's Work Effort", *Journal of Political Economy*, **82**, S1360-S163.

HECKMAN, J. J. (1979), "Sample Selection Bias as a Specification Error", *Econometrica*, **47**, 153-161.

HECKMAN, J. J. and MACURDY, T. E. (1980), "A Life Cycle Model of Female Labour Supply", *Review of Economic Studies*, **47**, 47-74.

KEMSLEY, W. F. F. (1977), "Family Expenditure Survey—A Study of Differential Response Based on a Comparison of the 1971 Sample with the Census", *Statistical News*, **31**, 16-22.

LATHAM, R. W. (1980), "Quantity Constrained Demand Functions", *Econometrica* **48**, 307-313.

LEE, L. F., MADALLA, G. S. and TROST, R. P. (1980), "Asymptotic Covariance Matrices of Two-Stage Probit and Two-Stage Tobit Methods for Simultaneous Equation Models with Selectivity", *Econometrica*, **48**, 491-503.

MACURDY, T. E. (1981), "An Empirical Model of Labour Supply in a Life Cycle Setting", *Journal of Political Economy*, **89**, 1059-1085.

MACURDY, T. E. (1982), "Interpreting Empirical Models of Labour Supply in an Intertemporal Framework with Uncertainty" (mimeo, Stanford University).

MACURDY, T. E. (1983), "A Simple Scheme for Estimating an Intertemporal Model of Labour Supply and Consumption in the Presence of Taxes and Uncertainty", *International Economic Review*, **24**, 265-289.

MUELLBAUER, J. (1983), "Surprises in the Consumption Function", *Economic Journal* (*conference papers*), 34-50.

NEARY, J. P. and ROBERTS, K. W. S., (1980), "The Theory of Household Behaviour Under Rationing", *European Economic Review*, **13**, 25-42.

SMITH, J. (1977), "Family Labour Supply over the Life Cycle", *Explorations in Economic Research*, **4**, 205-276.

SMITH. R. J. and BLUNDELL, R. W. (1983), 'An Exogeneity Test for a Simultaneous Equation Tobit Model with an Application to Labour Supply', (Discussion Paper in Econometrics ES152, University of Manchester), forthcoming *Econometrica*.

STERN, N. (1986), "On the Specification of Labour Supply Functions", in Blundell, R. W. and Walker, I. (eds.), *Unemployment, Search and Labour Supply* (Cambridge: Cambridge University Press, forthcoming).

TOBIN, J. (1958), "Estimation of Relationships for Limited Dependent Variables", *Econometrica*, **26**, 24-26.

WALES, T. J. and WOODLAND, A. D. (1980), "Sample Selectivity and the Estimation of Labour Supply Functions", *International Economic Review*, **21**, 417-467.