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MODELLING THE JOINT DETERMINATION OF HOUSEHOLD LABOUR SUPPLIES AND COMMODITY DEMANDS*

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In addition to choosing the allocation of total expenditure between commodities, households may also be able to make decisions over the allocation of their time between market work and leisure. In both theoretical and empirical work it has often been the case that these decisions have been analysed separately. In this paper we stress the theoretical attractions of considering the *joint* determination of the allocation of time between work and leisure and the allocation of total expenditure between commodities in a utility maximising framework. Using a sample of individual households we attempt to evaluate the empirical importance of the joint determination model over the separate determination of labour supplies and commodity demands. Our approach follows that of Abbott and Ashenfelter (1976), Phlips (1978), Barnett (1979), Deaton and Muellbauer (1980), and Atkinson and Stern (1980). Here we pay particular attention to the following four important aspects of household decision-making over commodity demand and labour supply.

The first concerns the commonly assumed restriction on the household's preferences of (weak) separability between goods and leisure. This assumption allows the estimation of commodity demand systems and Engel curves that exclude wage-rate variables. In cross-section household budget data with large variation in wage rates across the sample the invalidity of this assumption would involve a serious mis-specification. Any correlation between the excluded wage variable and the included price, income and demographic variables would lead to biased parameter estimates and hence biased estimated elasticities. Similarly, this assumption allows the estimation of labour supply curves that exclude relative price variables and its invalidity could produce biased labour supply elasticities.

A second aspect of household decision-making which we wish to highlight arises from the suspicion, commonly alluded to in labour economics texts, that primary male workers may not be free to choose their hours of work. Thus we estimate a matched pair of rationed and unrationed systems following the work of Neary and Roberts (1980) and Deaton and Muellbauer (1979). The separability restriction is not unrelated to this point, since without it we will see that the rationed hours of work enter demand systems in a complex fashion. In effect,

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in the absence of separability, rationing has direct substitution effects in addition to the indirect income effects.

The third aspect we wish to emphasise concerns the leisure decision. Here female leisure is considered as distinct from male leisure both with respect to commodity consumption and to its response to changes in household composition. Moreover, if male labour supply is predetermined because of rationing, the household's income will still be endogenous through its dependence on the female labour supply decision. Thus we regard it as important to model female labour supply behaviour despite the fact that it complicates the econometric analysis.

Finally, we introduce demographic variables to capture the effects of household composition on both labour supplies and commodity demands. We do this by extending the translation method of Pollak and Wales (1978), but other methods could equally be used, to incorporate the effect of household composition on leisure as well as commodity expenditures. The empirical significance of the effect of young children on female labour supply decisions has been demonstrated by Barton *et al.* (1980) and here we attempt to distinguish between the number of children and their ages by allowing for economies of scale. Estimates of the effects of household composition on expenditure have often been used to compute a cost of living sub-index defined over commodities alone and then used to make welfare comparisons between households (see Blundell (1980) and Muellbauer (1974)). However, Pollak and Wales (1979) point out that such a comparison ignores the utility which children yield to their parents. Indeed, if composition is endogenous, the logic of their argument suggests that children should leave the household's cost of living unaffected since the equivalent income generated by their presence must be at least as great as the extra expenditure entailed. The empirical issue of endogeneity we leave for subsequent research but note that when making welfare comparisons within the traditional framework the cost in terms of forgone female time available for work or leisure is likely to be at least as important as the cost in terms of commodity expenditures. In addition, a cost of living sub-index defined over commodities alone will only be defined when preferences are separable between goods and leisure (see Blackorby *et al.* (1978, p. 327)).

Since the separability assumption is crucial to the purpose of this paper, we choose a model which allows this assumption to be tested against the data. The *a priori* unreasonableness of the separability assumption can be demonstrated by noting that if all goods are normal and leisure is a substitute for at least one commodity then separability requires that it be a substitute for all commodities. This is an unattractive restriction to impose as a maintained hypothesis since at least some consumption activities are likely to be complementary to leisure. Thus, by choosing a model which does not impose separability we avoid these unattractive properties and we can discover which goods are complements to leisure and which are substitutes. The usefulness of such a classification stems from the role of complementaries between goods and leisure in optimal taxation theory. For example, if in a two good world, good 1 is a substitute for leisure and good 2 is a complement to leisure and the tax rates are initially the same on both goods, then by decreasing the tax rate on good 1 and increasing the tax rate on good 2, holding tax revenue constant, the supply of labour will increase. This is because taxing complements to leisure is effectively an indirect way of taxing leisure itself (see, for example, Atkinson and Stiglitz (1972) and Sandmo (1976)).

The assumption of separability has been rejected by Barnett (1979) using a time series of U.S. data. However, separability is a hypothesis about individual preferences, so it is clearly preferable to test it against micro data rather than against aggregate time series data. In this study we utilise family budget and labour supply data and the consequent large variation in wage rates across households should give greater power to our separability test.

As we have suggested, it is not clear that all leisure decisions of the household are freely made. Hours worked may be determined by employer preferences, union pressure, legislation or involuntary unemployment. Given the lack of information in our data set concerning the presence or absence of such constraints we allow for both possibilities in our estimation. When male labour supply is rationed all variability in household income arises from female labour supply behaviour. This contrasts significantly to the leisure-goods model of Atkinson and Stern (1980) where male labour supply is assumed to be unrationed and female labour supply has no direct effect on leisure-goods decisions. To estimate both rationed and unrationed models on an equal footing we require a matched pair of leisure-goods demand systems. Our model is a generalisation of the familiar Linear Expenditure System which, although implying quasi-homothetic preferences, no longer imposes the unrealistic additive separability restrictions and is the most general specification for which a matched pair of demand systems is easily obtainable.

Our data is a sample of two adult manual worker families drawn from the U.K. Family Expenditure Survey of 1974. All households selected contained married women and this raises the problem of modelling the female participation decision. The decision to participate or not depends on the difference between the offered wage and the value of time spent at home. Obviously this difference will depend on the female's personal characteristics, and on household composition, as well as on wages and prices. However, we cannot observe the offered wage for non-participating females so that our model is left with a limited dependent variable and lack of data on an explanatory variable when the limit is observed. Such a model is a generalisation of the limited dependent variable type of Tobin (1958). Wales and Woodland (1980) consider several possible estimation methods for such models. Since we are dealing with a system of utility maximising consistent demand equations it proves too difficult to integrate out the unobserved female wage variable. Further, we have few variables with which to predict female wage and so we select households with a participating female only correcting for the resulting selectivity bias using a technique due to Amemiya (1974) and Heckman (1979).

The paper proceeds as follows. Section I sets up the problem using constrained and unconstrained household cost (expenditure) functions in a way which emphasises the similarities between the problems of constraints on male leisure decisions and the implicit constraint on female leisure decisions due to their inability to choose more than T hours leisure, where T is the maximum number of hours available for work and leisure. That is, the participation decision is viewed as one of constrained decision making. A functional form is chosen for the cost function which allows for non-separability and enables us to estimate demand equations for a sufficient number of goods to make non-separability a possibility. Section II outlines the technique used to correct for the selection bias caused by the exclusion of households with non-participating females from the sample. Section III describes the data, discusses some further estimation problems, and presents the estimates. We conclude with a discussion of the merits of the estimates and some comments on directions for future research in Section IV.

I. SPECIFICATION OF THE MODEL

We suppose that each household faces the same prices $\mathbf{p} = (p_1, \ldots, p_n)$ for n goods $\mathbf{q} = (q_1, \ldots, q_n)$ and different marginal values $\mathbf{w} = (w_m, w_f)$ for their male and female leisure time $\mathbf{l} = (l_m, l_f)$. If T represents the total time available, and μ' is unearned income, then the household budget constraint may be written as

$$\mathbf{p}'\mathbf{q} + \mathbf{w}'\mathbf{l} = w_m T + w_f T + \mu' = \mu,$$

where μ is known as full income.

Assuming that the utility function for each household, $U(\mathbf{q},\mathbf{l})$, is strictly quasi-concave, then the minimised cost of attaining a given utility level $\overline{U}$ defines the full cost function.

$$C(\mathbf{p}, \mathbf{w}, \overline{U}) = \min_{\mathbf{q},\, \mathbf{l}} [\mathbf{p}'\mathbf{q} + \mathbf{w}'\mathbf{l} \mid U(\mathbf{q},\mathbf{l}) \geq \overline{U}]. \tag{1}$$

The form of (1) chosen for our analysis is a generalisation, due to Muellbauer (1981), of the Gorman Polar form given by

$$C(\mathbf{p}, \mathbf{w}, \overline{U}) = a(\mathbf{p}) + w_m d_m(\mathbf{p}) + w_f d_f(\mathbf{p}) + b(\mathbf{p})^{1-\theta_f-\theta_m} w_f^{\theta_f} w_m^{\theta_m} \overline{U}, \tag{2}$$

where $a(\mathbf{p})$ and $b(\mathbf{p})$ are concave linear homogeneous functions and $d_m(\mathbf{p})$ and $d_f(\mathbf{p})$ are homogeneous of degree zero. The form of (2) is convenient since we can derive an explicit functional form for its rationed counterpart. In addition, the system of demand equations derived from (2) allows separability between goods and leisure to be tested. As with the Linear Expenditure System the interpretation of the first three terms in (2) is the necessary expenditures; in this case, necessary expenditures out of full income on commodities, male leisure and female leisure respectively. Notice that the necessary quantities are not assumed to be constants as in the Linear Expenditure System but are allowed to be general functions of prices. Since, however, our data are derived from a single cross section so that all commodity prices are constant the particular functional forms of $a(\mathbf{p})$, $d_m(\mathbf{p})$, $d_f(\mathbf{p})$ and $b(\mathbf{p})$ are not important provided they satisfy the homogeneity conditions. Indeed, they could be any general second-order flexible forms and therefore we are not imposing any separability restrictions between groups of commodities.

For purposes of exposition however, it is convenient to choose the following functional forms. We let the necessary costs of commodities, $a(\mathbf{p})$, have the form

$$a(\mathbf{p}) = \sum_{i=1}^{n} p_i \gamma_i m_i,$$

where m_i are the number of equivalent adults in each household with respect to good i. For necessary male leisure hours we specify

$$d_m(\mathbf{p}) = \gamma_m \prod_{i=1}^{n} p_i^{\delta_{mi}}, \quad \gamma_m > 0, \quad \sum_i \delta_{mi} = 0$$

and for necessary female leisure we add a term m_f which captures the direct effect of household composition on female leisure. Thus

$$d_f(\mathbf{p}) = \gamma_f \prod_{i=1}^{n} p_i^{\delta_{fi}} + m_f, \quad \gamma_f > 0, \quad \Sigma\delta_{fi} = 0$$

$$= \bar{d}_f(\mathbf{p}) + m_f.$$

Finally, we write

$$b(\mathbf{p}) = \prod_{i=1}^{n} p_i^{b_i}, \quad \sum_i b_i = 1.$$

Notice that household composition enters only via necessary commodity expenditures $a(\mathbf{p})$, and necessary female leisure time $d_f(\mathbf{p})$ (we assume that there is no direct effect of composition on the necessary male leisure time requirement). This method of incorporating household composition is an extension of the translation approach of Pollak and Wales (1978) to the leisure-goods model, and is explored in more detail in Blundell (1980). The precise specification and interpretation of m_i and m_f are left until Section III.

The compensated commodity and leisure demand equations are obtained from the price and wage derivatives of (2). Eliminating $\bar{U}$ from these equations using (2) yields both the system of uncompensated demand equations given by

$$p_i q_i = p_i \gamma_i m_i + \delta_{mi} w_m d_m + \delta_{fi} w_f d_f + (1-\theta_m-\theta_f) b_i [\mu' + (T-d_f) w_f + (T-d_m) w_m - a], \quad (3)$$

and, letting $h = (T-l)$ denote hours worked, the following labour supply equations

$$w_f h_f = w_f(1-\theta_f)(T-d_f) - \theta_f[\mu' + (T-d_m) w_m - a], \quad (4)$$

$$w_m h_m = w_m(1-\theta_m)(T-d_m) - \theta_m[\mu' + (T-d_f) w_f - a]. \quad (5)$$

Since prices are constant across households the terms d_m, $\bar{d}_f$ and b_i can be estimated as single parameters.

Turning to the separability issue, the results of Goldman and Uzawa (1964, p. 392) imply that for a group of commodities I to be separable from good $j \notin I$ requires

$$C_{ij} = kC_{iu} \quad \text{for all} \quad i \in I,$$

where k is some constant function for all i, and the subscripts refer to the derivatives of the cost function. In the context of our leisure-goods model, separability of goods from male leisure requires

$$\delta_{mi} d_m = k b_i \quad \text{for all} \quad i = 1, \ldots, n. \tag{6}$$

However, the restrictions $\Sigma b_i = 1$ and $\Sigma \delta_{mi} = 0$ forces k to be zero so that (6) becomes

$$\delta_{mi} = 0 \quad \text{for all} \quad i = 1, \ldots, n.$$

Similarly, separability of goods from female leisure requires

$$\delta_{fi} = 0 \quad \text{for all} \quad i = 1, \ldots, n.$$

Inspection of our commodity demand system given by (3) shows that the omission of w_m and w_f would imply separability of goods from both male and female leisure. Thus our chosen cost function allows a particularly simple test of separability and, moreover, the test is not data dependent.

Turning now to the problem posed by constraints on male labour supply we write the cost minimising problem for the household subject to a given male labour supply as

$$C^R(\mathbf{w}, \mathbf{p}, \bar{l}_m, \bar{U}) = \min_{\mathbf{q},\, l_f} (w_m \bar{l}_m + w_f l_f + \mathbf{p}'\mathbf{q} \mid U \geqslant \bar{U}). \tag{7}$$

C^R is the minimum cost of achieving $\bar{U}$ at prices w_m, w_f and $\mathbf{p}$ given that the male is constrained to take exactly $\bar{l}_m$ leisure hours. The properties of the rationed cost function (7) are described in detail by Neary and Roberts (1980) and Deaton and Muellbauer (1980). For our purposes it is sufficient to outline the relationship between the rationed and unrationed cost functions and to derive the system of rationed demand equations.

Since $\bar{l}_m$ is not a choice variable we can define the conditional cost function as

$$\begin{aligned} \tilde{C}(w_f, \mathbf{p}, \bar{l}_m, \bar{U}) &= \min_{\mathbf{q},\, l_f} (w_f l_f + \mathbf{p}'\mathbf{q} \mid U \geqslant \bar{U}) \\ &= C^R - w_m \bar{l}_m. \end{aligned} \tag{8}$$

To relate the rationed and unrationed functions we define the *virtual* wage, $\bar{w}_m$, to be that wage which would just induce the ration to be freely chosen; that is

$$\frac{\partial C(w_f, \bar{w}_m, \mathbf{p}, \bar{U})}{\partial w_m} = \bar{l}_m. \tag{9}$$

As shown in Deaton and Muellbauer (1979) the convenience of our chosen cost function is that $\bar{w}_m$ can be solved for explicitly from (9). At $\bar{w}_m$ the minimum cost of achieving $\bar{U}$ is the same whether the ration is imposed or not; that is,

$$C(w_f, \bar{w}_m, \mathbf{p}, \bar{U}) = C^R(w_f, \bar{w}_m, \mathbf{p}, \bar{l}_m, \bar{U}). \tag{10}$$

We can rewrite (10) in terms of the conditional cost function, yielding

$$C(w_f, \bar{w}_m, \mathbf{p}, \bar{U}) = \tilde{C} + \bar{w}_m \bar{l}_m,$$

which after eliminating $\tilde{C}$ using (8) gives

$$C(w_f, \bar{w}_m, \mathbf{p}, \bar{U}) = C^R - \bar{l}_m(w_m - \bar{w}_m),$$

and rearranging gives

$$C^R = \bar{C} + \bar{l}_m(w_m - \bar{w}_m), \tag{11}$$

where $\bar{C}$ is the unrationed cost function, C, evaluated at the virtual wage $\bar{w}_m$. From the unrationed cost function (2) we can use (9) to derive the following expression for the virtual wage

$$\bar{w}_m = \left(\frac{\theta_m b^{1-\theta_f-\theta_m} w_f^{\theta_f} \bar{U}}{\bar{l}_m - d_m}\right)^{1/(1-\theta_m)}. \tag{12}$$

Substituting (12) into (11) gives our functional form for the rationed counterpart to (2) as

$$C^R = a + d_f w_f + \bar{l}_m w_m + b^{1-\rho_1} w_f^{\rho_1} (\bar{l}_m - d_m)^{-\rho_2} \theta_m^{\rho_2} (1-\theta_m)\, \bar{U}^{1/(1-\theta_m)}, \tag{13}$$

where $\rho_1 = \theta_f/(1-\theta_m)$ and $\rho_2 = \theta_m/(1-\theta_m)$. The system of rationed demand equations are obtained from the derivatives of C^R with respect to p_i giving

$$p_i q_i = p_i \gamma_i m_i + d_f \delta_{fi} w_f + [(1-\rho_1) b_i + \rho_2 d_m \delta_{mi}/(\bar{l}_m - d_m)] \times [\mu' + w_f(T - d_f) + w_m(T - \bar{l}_m) - a]. \tag{14}$$

Separability of goods from male and female leisures still requires that $\delta_{mi} = 0$ and $\delta_{fi} = 0$ for all i. Notice that unless goods are separable from male leisure, the ration affects not only the level of full supernumerary income (the final term in square brackets in (14)) but also each marginal propensity to consume (the first term in square brackets in (14)). Thus, for example, if a male worker is rationed to work less than he desires, full income is reduced, *ceteris paribus*, which with constant marginal propensities would simply have income effects on expenditures. But, in the absence of separability, the rationed worker will substitute away from substitutes to leisure into complements to leisure, that is he reduces his marginal propensities to consume substitutes to leisure and increases his marginal propensities to consume complements to leisure.

The problem of non-participating females is now easily incorporated into the analysis by regarding non-participation as the constraint $l_f = T$. When $l_f = T$ we can write the rationed cost function corresponding to (11) as

$$C^R(w_f, w_m, \mathbf{p}, T, \bar{U}) = C^* + T(w_f - w_f^*),$$

where $C^* = C(w_m, w_f^*, \mathbf{p}, \bar{U})$ is the unrationed cost function evaluated at w_f^*, the virtual price of female leisure at which $l_f = T$ would be freely chosen. This wage is more commonly referred to as the reservation wage. Females will choose to participate if the cost of doing so is less than the cost of not participating; that is, if $C^* < C^R$. Thus participation occurs when the offered wage, w_f, is greater than the reservation wage, w_f^*.

II. ECONOMETRIC ANALYSIS

As is usually the case in demand analysis, all equations in either the unrationed or rationed system contain the same explanatory variables. In addition, since both systems satisfy the usual adding up restrictions exactly, we can delete one equation from each system without loss of information. To describe the estimation method used in this study we concentrate on the unrationed system (3) and (4) and delete equation (5), but exactly the same technique is used on the rationed system (14). We write a stochastic version of (3) and (4) as

$$Y_i = \mathbf{X}'\boldsymbol{\beta}_i + \epsilon_i \quad (i = 1, \ldots, n, f), \tag{15}$$

where ϵ_i is normally distributed with zero mean and constant variance σ_i^2. Expression (15) is a system of $n+1$ expenditure and female earnings equations where the $\boldsymbol{\beta}_i$ terms are nonlinear functions of the underlying parameters and for $i = 1, \ldots, n$, $Y_i = p_i q_i$, while for $i = f$, $Y_f = w_f h_f$. All dependent variables in this, or any other, expenditure system are constrained to be non-negative but, as we will show, this is unimportant provided the probability of attaining the zero limit is very small. However, for female labour income Y_f, this is not the case since a zero dependent variable will be observed whenever the offered wage is less than the reservation wage.

Estimating a system like (15) using joint least squares on a selected sample where $Y_f > 0$ gives rise to inconsistency. This can be seen from examining the expectation of the disturbances given $Y_f > 0$. For female participants we have

$$\mathrm{E}(\epsilon_f | Y_f > 0) = \mathrm{E}(\epsilon_f | \epsilon_f > -\mathbf{X}'\boldsymbol{\beta}_f) \neq 0, \tag{16}$$

and similarly

$$\mathrm{E}(\epsilon_i | Y_f > 0) = \mathrm{E}(\epsilon_i | \epsilon_f > -\mathbf{X}'\boldsymbol{\beta}_f) \neq 0 \quad \text{for all} \quad i \neq f, \tag{17}$$

provided $\mathrm{E}(\epsilon_i \epsilon_f) \neq 0$. Assuming normality for the disturbances the probability that $\epsilon_f > -\mathbf{X}'\boldsymbol{\beta}_f$ is given by $1 - G(L)$, where $L = -\mathbf{X}'\boldsymbol{\beta}_f/\sigma_f$ and G is the cumulative normal density function. Following Tallis (1961) the conditional expectations in (16) and (17) can now be written as

$$\mathrm{E}(\epsilon_i | \epsilon_f > -\mathbf{X}'\boldsymbol{\beta}_f) = \lambda \sigma_{if}/\sigma_f \quad \text{for all} \quad i = 1, \ldots, n, f,$$

where $\lambda = g(L)/[1 - G(L)]$, g being the standard normal density function.

By including λ linearly in all equations in (15) we can obtain consistent estimates using joint least squares. Unfortunately λ is unknown unless $\boldsymbol{\beta}_f/\sigma_f$ is known. In this study we estimate λ by first estimating $\boldsymbol{\beta}_f/\sigma_f$ using an instrumental variable estimator due to Amemiya (1973).

This ingenious estimator is derived from squaring equation (15) and taking expectations conditional on the female participating to give

$$\mathrm{E}(Y_f^2 | \epsilon_f > -\mathbf{X}'\boldsymbol{\beta}_f) = \mathbf{X}'\boldsymbol{\beta}_f \mathrm{E}(Y_f | \epsilon_f > -\mathbf{X}'\boldsymbol{\beta}_f) + \sigma_f^2.$$

Thus we estimate

$$Y_f^2 = Y_f \mathbf{X}'\boldsymbol{\beta}_f + \sigma_f^2 + \eta,$$

where $\eta = \epsilon_f^2 - \sigma_f L \epsilon_f - \sigma_f^2$, using the instruments $(\mathbf{X}\hat{Y}_f \colon 1)$, where $\hat{Y}_f$ is the least-squares prediction from (15). This consistently estimates $\boldsymbol{\beta}_f$ and σ_f^2, from which we can derive $\hat{\boldsymbol{\beta}}_f/\hat{\sigma}_f$ and finally

$$\hat{\lambda} = \frac{g(\mathbf{X}'\hat{\boldsymbol{\beta}}_f/\hat{\sigma}_f)}{1 - G(-\mathbf{X}'\hat{\boldsymbol{\beta}}_f/\hat{\sigma}_f)}.$$

While it is true that the properties of this estimator are heavily dependent on the normality assumption, this is also the case for all alternative procedures such as the probit method of Heckman (1979).

The introduction of $\hat{\lambda}$ into each equation of the system (15) produces consistent estimates of all parameters when joint least squares is used on the selected sample. The variance-covariance matrix for the resulting estimates of the underlying preference parameters can be obtained from an extension of the results of Lee *et al.* (1980) and its derivation is available from the authors on request.

III. EMPIRICAL RESULTS

Only households containing two married adults of working age with the head of household a male manual employee were selected. Choosing a sample with working wives reduced the number of households from 208 to 115 implying a participation rate (in this sample of working age females) of a little over 55 %. Following the procedure of Atkinson and Stern (1980) the marginal wage rates were calculated by multiplying the normal gross hourly earnings by one minus the basic tax rate which includes an adjustment for national insurance contributions. Unearned income was then defined simply by the linear budget constraint. In order to reduce the importance of unexpected fluctuations in hours worked, normal rather than actual hours works were used for both males and females.

The form of our cost function (2) assumes quasihomothetic preferences and in order to make this assumption more palatable we made a further selection of households, choosing those with total weekly expenditure (on all goods except housing) in the range of £35 to £55 per week, the pre-selection sample average being £42.60. The resulting sample contained 103 observations. This selection on the basis of a sum of dependent variables will, in general, lead to inconsistent parameter estimates. However, as 90 % of the sample fell within this expenditure range the resulting inconsistency is likely to be small, and for this reason we only correct for the selection bias caused by the selection of female participants. Finally to reduce the possibility of heteroskedasticity all dependent variables were defined as expenditure shares.

The effect of household composition on the leisure-goods choices of the household was entered via the necessary expenditure terms of our cost functions as described in Section I. The commodity specific composition effects, m_i, were allowed to be general continuous functions of household age structure and size, the estimated parameters of which give the underlying continuous commodity equivalence scales as discussed in Blundell (1980). These commodity specific effects have an *indirect* effect on labour supply through the function $a(\mathbf{p})$ which enters the supernumerary full income terms in the labour earnings equations. An increase in the number or age of children tends to increase $a(\mathbf{p})$ and have a positive effect on both male and female labour supply (if male and female leisure are normal goods). We also allow composition to enter female labour supply decisions *directly* through the term m_f in the $d_f(\mathbf{p})$ equation. Although there are many possible ways of entering composition, this is not the primary aim of this paper and we choose a particularly parsimonious form which allows for age and scale effects. In particular we write

$$m_f = \gamma_a n + \gamma_b n^2,$$

where $n = \Sigma_{t=0}^{18} \alpha^t n_t$ and where n_t is the number of children of age t and α is a parameter which captures the depreciating effect of age of child on female labour supply. The parameter α was estimated by grid search across the range 0·5–1·0. While we recognise that a continuously declining age effect will not capture perfectly the structural changes due to children attending school with varying day lengths as they get older, we feel that it should act as a good approximation.

Table 1

*Parameter Estimates of the Unrationed Leisure-Goods Model**

Commodity group	γ_i	$(1-\theta_m-\theta_f)\,b_i$	$d_m\delta_{mi}$	$\bar{d}_f\delta_{fi}$	σ_{fi}/σ_f
Food	4·8198	0·0613	−0·7809	−1·1603	0·0484
	(0·9261)	(0·0401)	(1·2011)	(1·7758)	(0·0458)
Energy	0·9261	0·0021	−0·6302	1·4301	0·0251
	(0·2311)	(0·0148)	(0·7491)	(0·7654)	(0·0182)
Clothing	0·9721	0·0813	−0·9496	2·5208	−0·0201
	(0·3628)	(0·0357)	(1·1666)	(1·0566)	(0·0408)
Durables	1·3857	0·0628	−2·8993	−0·8381	0·0817
	(0·6243)	(0·0312)	(1·0681)	(1·2879)	(0·0357)
Transport	2·0103	0·0621	2·6688	1·3184	0·0853
	(0·7285)	(0·0373)	(1·2011)	(1·8795)	(0·0542)
Services	2·6200	0·1465	2·5914	−4·9470	−0·0306
	(1·0586)	(0·0472)	(1·2684)	(2.1302)	(0·0466)

$T-d_f$ = 36·5865	θ_f = 0·2212	γ_a = 13·9176	α = 0·9	χ^2_m = 19·16
(2.4952)	(0·0177)	(2·9591)		
$T-d_m$ = 47·6861	$1-\theta_f-\theta_m$ = 0·4159	γ_b = −3.1871	σ_f = 0·1070	χ^2_f = 22·13
(3·1185)	(0·0586)	(1·0407)	(0·0465)	

* Asymptotic standard errors in parentheses. The critical $\chi^2_{0.01}$ value for 5 degrees of freedom is 15·09.

This direct influence of composition on female labour supply will tend to counteract the indirect expenditure effect that enters through $a(\mathbf{p})$. We would expect the direct (indirect) effect to dominate for households with younger (older) children and hence lead to a decrease (increase) in female labour supply. The interplay between these composition effects is obviously of some interest for horizontal equity and provides another reason for analysing expenditure and leisure decisions simultaneously.

The estimates of the unrationed model with the correction for selection bias are given in Table 1. The nonlinear routine RESIMUL of Wymer (1973) was used to generate the parameter estimates having deleted the male labour supply equation in order to remove the singularity of the system. Since all parameters automatically satisfy the adding-up restrictions across equations the estimates are invariant to the equation deleted.

Overall the parameters look plausible. We note that the crucial restriction of separability between goods and leisure, for which Wald test statistics are given by χ_f^2 and χ_m^2, can be rejected.[1] To consider the importance of complementarities between goods and leisure it is useful to note that for our cost function the compensated substitution effects between good i and, say, female leisure is given by

$$C_{if} = \delta_{if} d_f + \theta_f C_{iu} \bar{U}.$$

Table 2

*Parameter Estimates of the Rationed Leisure-Goods Model**

Commodity group	γ_i	$(1-\rho_1) b_i$	$\rho_2 d_m \delta_{mi}$	$\bar{d}_f \delta_{fi}$	σ_{fi}/σ_f
Food	3·3696 (0·6032)	0·1512 (0·0313)	−0·1076 (0·0567)	−0·1842 (0·4641)	−0·0976 (0·0516)
Energy	0·7109 (0·2551)	0·0150 (0·0129)	−0·0233 (0·0164)	0·8247 (0·6107)	0·0148 (0·0216)
Clothing	0·7777 (0·6266)	0·1114 (0·0284)	0·0137 (0·0401)	1·3065 (0·6015)	−0·2117 (0·0499)
Durables	1·4496 (0·4560)	0·0976 (0·0258)	−0·0263 (0·0521)	−1·1148 (0·2435)	0·0244 (0·0446)
Transport	2.0420 (0·9602)	0·1821 (0·0307)	−0·0800 (0·0454)	0·9311 (1·5401)	−0·0289 (0·0521)
Services	1·3911 (0·8649)	0·2491 (0·0366)	0·2195 (0·0852)	−1·7633 (1·0613)	−0·1265 (0·0620)

$T-\bar{d}_f = 40·8869$ (2·7300) $\quad 1-\rho_1 = 0·8066$ (0·0410) $\quad \gamma_a = 20·7869$ (3·6308) $\quad \alpha = 0·9 \quad \chi_m^2 = 18·65$

$\gamma_b = -4·3862$ (1·0781) $\quad \chi_f^2 = 21·01$

* Asymptotic standard errors in parentheses. The critical $\chi_{0.01}^2$ value for 5 degrees of freedom in 15·09.

Since θ_f, and $\bar{U}$ are both positive, as is C_{iu} if female leisure is a normal good, a strongly negative estimate of $\delta_{if} d_f$ indicates complementarity, while a positive $\delta_{if} d_f$ indicates substitutability. Services and transport are strong substitutes for male leisure, whereas clothing, food, energy and our definition of durables tend to be complements to male leisure[2]. As might be expected these goods do not necessarily have the same relationships with female leisure. Services tend to be complementary to female leisure, clothing is a substitute and energy tends to be a complement. The presence of children in the household has a pronounced effect on female labour supply, since γ_a is highly significant and large, the birth of a first child reducing the time available for female work by nearly 14 hours per week. γ_b is significantly negative indicating economies of scale in the care of children.

The estimates of the parameters of the rationed model are presented in

[1] Using a similar data set, but without explicitly modelling female labour supply, Blundell (1980) was unable to reject the separability of goods from male leisure. This indicates the crucial importance of female leisure in the rejection of separability.

[2] Energy covers fuel, light and power, clothing includes footwear; transport includes vehicles; we have excluded housing expenditure, implicitly treating it as separable from all other decisions, and we treat expenditure on durables as current consumption. Note that the durables definition includes items which are 'time saving' as well as 'time using', so that it would be desirable to disaggregate this group.

Table 2. Again the Wald tests indicate the rejection of separability which implies that even if labour income is given it is not sufficient to let leisure time effect commodity demands simply through the income term. The estimate of the labour time available for females, $T-\bar{d}_f$, has increased over the unrationed estimate such that now all females in the sample work less than this maximum. The relationships between goods and female leisure remains unchanged, but transport now tends to be a complement to male leisure. The effect of children on female labour supply is now even more pronounced and economies of scale are greater.

Table 3

Labour Supply Elasticities

	No. of children	Unrationed	Rationed
Male		−0·2863	
	0	0·4274	0·6489
Female	1	0·1074	0·0889
	2	−0·1926	−0·3010

Finally it is important to note that the unrationed and rationed models cannot be compared statistically since the two systems contain different numbers of dependent variables and have quite different stochastic specifications.[1] Without further information on the degree of rationing in the sample (see, for example, Ham (1981)) it is difficult to test the rationing hypothesis. The most that can be said from a comparison of Table 2 with Table 1, is that there are no indications that rationing is not at work. More positive conclusions await improved data.

Since we are working with a single cross-section exhibiting no price variation, we are unable to identify the own price elasticities for goods. However, wages do vary across the sample and we are able to identify the labour supply elasticities and evaluate these at the average hours worked (39·6 per week for males, 20·2 for females). For the unrationed case these are given by

$$\frac{w_j}{h_j}\frac{\partial h_j}{\partial w_j}=\frac{(1-\theta_j)\,(T-d_j)}{h_j}-1 \quad (j=m,f),$$

and for the rationed model, the female labour supply elasticity is given by

$$\frac{w_f}{h_f}\frac{\partial h_f}{\partial w_f}=\frac{(1-\rho_1)\,(T-d_f)}{h_f}-1.$$

Table 3 presents these elasticities for no children, for one child aged 3 years, and for two children aged 3 and 6 years. Notice that for zero children the data here are consistent with much previous evidence of backward bending male labour supply and forward sloping female labour supply. As we would intuitively expect the labour supply of a female with a small child becomes more inelastic as it becomes more difficult to substitute into market time. But with two children the income

[1] It is simple to demonstrate that if we let the unrationed model have additive homoskedastic disturbances and integrate these back into the cost function, then the corresponding rationed model will have neither additive nor homoskedastic disturbances.

effect begins to dominate the substitution effect and the female labour supply elasticity becomes negative. Contrasting the rationed and unrationed elasticities we can see some evidence that, in the presence of constraints on male labour supply, female labour supply becomes more responsive to changes in the female wage.

IV. CONCLUSIONS

This paper has attempted to analyse the interactions between household decisions over labour supplies and commodity demands in a utility maximising framework. These decisions have been modelled under the assumptions that either male labour supply is freely chosen or the observed male hours of work are exogenously determined so effectively imposing a ration on the household. We demonstrated the importance of the separability assumption in this framework and tested the restrictions implied by separability using data from the U.K. Family Expenditure Survey. Under both the unrationed and rationed assumptions these restrictions were rejected.

The incorporation of female labour supply into the analysis required that we face the important problem of non-participation. For estimation purposes we selected a sample of households with participating females only and then corrected for the resulting selectivity bias. We found evidence of strong household composition effects on female labour supply entering both through necessary female leisure time and through necessary commodity expenditures. These effects would typically result in higher (lower) levels of participation for females with older (younger) children compared with a household with no children. We also found evidence of economies of scale in the rearing of children.

The results of this work suggest a number of interesting avenues for further research. The implications of rejecting separability for optimal taxation theory are clear, but the computation of an optimal taxation system requires estimates of all the parameters of the model which needs data from at least two cross sections for identification. Secondly we have shown that rationing may be important for household decision making through its income effects, and, given the rejection of separability, through its effects on marginal propensities to consume. Our empirical work has been only suggestive and further information on labour supply constraints will facilitate the pooling of our rationed and unrationed models. In principle, for example, our methods could be used to analyse involuntary unemployment along the lines pursued by Ashenfelter (1980) using times series data. Finally, we noted the importance of household size and composition for expenditure and leisure decisions. It would be particularly useful to attempt to incorporate composition and size effects into a lifecycle model since, if intertemporal separability is not true, our estimates may be picking up lifecycle phenomena. Moreover, we have assumed that children are exogenous and we may have overestimated the 'cost' of children to the extent that fertility decisions are part of a lifecycle plan.

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