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TAX REFORM AND WELFARE MEASUREMENT: DO WE NEED DEMAND SYSTEM ESTIMATION?*

James Banks, Richard Blundell and Arthur Lewbel

The exact measurement of the welfare costs of tax and price reform requires a detailed knowledge of individual preferences. Typically, first order approximations of welfare costs are calculated avoiding detailed knowledge of substitution effects. We derive second order approximations which, unlike first order approximations, require knowledge of the distribution of substitution elasticities. This paper asks to what extent simple approximations can be used to measure the welfare costs of tax reform and evaluates the magnitude of the biases for a plausible size tax reform. In our empirical examples first order approximations display systematic biases; second order approximations always work well.

This paper investigates the accuracy of simple welfare measures used to calculate the welfare costs of tax or price reforms. By simple measures we mean those that require limited access to individual level data and minimal knowledge of individual substitution effects. When every household's demands for goods are known (as functions of prices and income), the welfare effects of a price or tax change can be directly calculated. In practice, the demands of individual agents are not known so only approximate welfare measures are possible. This paper reviews the standard first-order approximations that are widely used in the literature, derives corresponding second-order approximations and investigates the accuracy of these approximations. The literature regarding first-order approximations to tax and price changes is based around the work of Feldstein (1972) and Stern (1987). Recent applications include Newbery (1995) for price changes in the United Kingdom and Hungary and Mayshar and Yitzhaki (1995) for an analysis of 'Dalton-improving' UK tax reform.

In many cases individual level data on demands (that are representative of the whole population) are rare. Reliable estimates of substitution effects are even more difficult to find. However, using standard results from consumer theory, it is well known that first-order approximations to welfare measures simply require a knowledge of demands themselves and not substitution effects (see Varian (1978, p. 411), for example). Moreover, if social utility weights are equal then aggregate demand will suffice. Second order approximations, which are derived here, require substantially more information. They depend on the distribution of substitution elasticities which requires estimates of the derivatives of demand functions. If social utility weights are equal then average elasticities are sufficient; otherwise social utility depends on a weighted average of price derivatives.

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Typically, the price or tax changes that are of the greatest policy interest are those involving substantial rather than marginal changes in price. In these cases substitution effects can be non-trivial. The marginal (i.e. first order) approximations ignore these effects, and therefore, can be seriously biased. To our knowledge, the magnitude of this bias has not been examined before. We do so here, and discuss its importance both theoretically and in the context of an indirect tax reform that adds a new group of goods to the expenditure tax base. Using a plausible description of preferences over broad commodity groups we find that, with indirect tax rates in Europe of between 10% and 20%, the bias is of the order of 5% to 10%. For a smaller, but still non-marginal, tax reform we show that a suitable choice of first order approximation can be used to yield only small errors in the measurement of welfare changes without a knowledge of individual substitution elasticities. However, in our empirical example the second-order approximation uniformly produces an improvement in the measurement of changes in aggregate social welfare.

I. APPROXIMATE WELFARE MEASURES

In what follows we examine the issue of welfare measurement at both the individual level, using money metric measures of welfare loss, and at the 'macro' level using a social welfare function to aggregate individual welfare levels. The latter requires information on individual preferences and individual utility weights in social welfare.

Define the social welfare function

$$\mathbf{U} = U(u_1, \dots, u_H) = U[v_1(x_1, p), \dots, v_H(x_H, p)] \quad (1)$$

over households $h = 1, \dots, H$, where u_h is the attained utility level of household h , which equals the indirect utility function $v_h(x_h, p)$ of household h having total expenditures x_h and facing price p for the good or service under analysis. We assume throughout that all consumers face the same price p . The indirect utility function also depends on the prices of other goods and may in addition depend on attributes of the household such as demographic characteristics. These other prices and variables are held constant throughout the analysis and so are not included here for notational simplicity. Let $q_h = q_h(x_h, p)$ denote the quantity of the good purchased by household h , expressed as Marshallian demands, that is, as a function of prices and total expenditures.

As in Stern (1987, p. 54), for each household h we can use (1) to define a social marginal utility of income weight θ_h :

$$\theta_h = \frac{\partial U[v_1(x_1, p), \dots, v_H(x_H, p)]}{\partial v_h(x_h, p)} \frac{\partial v_h(x_h, p)}{\partial x_h}, \quad (2)$$

Note that θ_h is implicitly a function of p and of x_1, \dots, x_H .

Abstracting from government revenue considerations¹, the effect on social

¹ For the purposes of this paper we will treat a tax reform as a proportional change in prices. The welfare results will therefore apply directly if the tax reform is revenue neutral. If government revenue changes, however, the change in social welfare will have an extra component depending on change in demand. The second order results we present in this paper can be used to compute this extra component.

welfare \mathbf{U} of an increase in price (which could correspond to a tax on quantities) from p to p^* ($= p + \Delta p$) is given by

$$\frac{\Delta \mathbf{U}}{\Delta p} = \frac{U[v_1(x_1, p^*), \dots, v_H(x_H, p^*)] - U[v_1(x_1, p), \dots, v_H(x_H, p)]}{\Delta p}. \quad (3)$$

For a small change in price this change in utility can be approximated by

$$\frac{\partial \mathbf{U}}{\partial p} = \sum_n \frac{\partial U[v_1(x_1, p), \dots, v_H(x_H, p)]}{\partial v_n(x_n, p)} \frac{\partial v_n(x_n, p)}{\partial p} = -\sum_n \theta_n q_n. \quad (4)$$

The last equality above follows from the definition of the utility weights θ_n and Roy's identity, and is commonly used to evaluate the social welfare effects of a price or tax change without explicitly estimating demand or indirect utility functions for individual households.

In the case where all the utility weights equal one, the change in utility just equals the total quantity of the good purchased times the change in price. Intuitively, one expects that this is an overestimate of the effect on welfare, because it equals the effect that would arise from the price change if consumers did not reduce their consumption of the good in response to the price rise. We will show this formally below.

The first order approximation

$$\Delta \mathbf{U} / \Delta p \approx \partial \mathbf{U} / \partial p = -\sum_n \theta_n q_n$$

can be replaced by a more accurate second order approximation using the Taylor expansion

$$\frac{\Delta \mathbf{U}}{\Delta p} \approx \frac{\partial \mathbf{U}}{\partial p} + \frac{\Delta p}{2} \frac{\partial^2 \mathbf{U}}{\partial p^2}. \quad (5)$$

Differentiating equation (4) by p gives

$$\frac{\partial^2 \mathbf{U}}{\partial p^2} = -\sum_n \left(\frac{\partial \theta_n}{\partial p} q_n + \frac{\partial q_n}{\partial p} \theta_n \right) \quad (6)$$

and combining the above equations yields the second order approximation

$$\frac{\Delta \mathbf{U}}{\Delta p} \approx -\sum_n \theta_n q_n \left[1 + \frac{\Delta p}{2p} \left(\frac{\partial \ln \theta_n}{\partial \ln p} + \frac{\partial \ln q_n}{\partial \ln p} \right) \right], \quad (7)$$

where the partial derivatives are evaluated at pre-reform prices.

Equation (7) shows that a second order correction to the usual approximation depends on the price elasticities of both the Marshallian demands and of the utility weights. It will be seen in the next section that sensible utility weights depend on prices except under very special circumstances, and therefore, the price elasticity of θ_n in equation (7) is generally non-zero.

Higher order approximations would involve second and higher derivatives of θ_n and q_n with respect to p , so the less the price elasticities of these variables vary with price, the better will be the quality of the second order approximation. Similarly, the less θ_n and q_n themselves vary with prices, the better will be the standard first order approximation.

Sensible utility weights are positive, purchased quantities are non-negative, and own price elasticities are negative (except in the peculiar case of Giffen goods), so unless the price elasticities of the utility weights are positive and large, the second order approximation has a smaller absolute magnitude than the first order approximation. This means that, assuming third and higher order terms in the expansion are small, the standard marginal first order approximation will systematically over estimate the social welfare effect of a non-marginal price or tax change, and the second order term acts to correct this bias.

It is often convenient in applied demand estimation to work with log prices and budget shares instead of level prices and quantities. Applying the same methods as above, this yields the slightly different first and second order approximations

$$\frac{\Delta U}{\Delta p} = \frac{\Delta \ln p}{\Delta p} \frac{\Delta U}{\Delta \ln p} \approx \frac{\Delta \ln p}{\Delta p} \frac{\partial U}{\partial \ln p} = -\frac{\Delta \ln p}{\Delta p} \sum_n \mu_n w_n \quad (8)$$

and

$$\frac{\Delta U}{\Delta p} \approx -\frac{\Delta \ln p}{\Delta p} \sum_n \mu_n w_n \left[1 + \frac{\Delta \ln p}{2} \left(\frac{\partial \ln \mu_n}{\partial \ln p} + \frac{\partial \ln w_n}{\partial \ln p} \right) \right], \quad (9)$$

where $w_n = q_n p / x_n$ is the budget share of the good and $\mu_n = \theta_n x_n$ is an alternative utility weight measure. This log form approximation is an expansion based on $\ln p$ instead of p , so it numerically differs from the level form, and the relative quality of the approximations will depend on the particular form of social welfare and individual utility functions. Note, however, that while the first order approximation in level form generally overestimates the welfare effects of a price change (because own price elasticities of quantities are almost always negative) the same systematic bias need not be present in the log form approximation.

II. UTILITY WEIGHTS

In virtually all tax or policy applications of the welfare approximation (4) utility weights are treated as fixed constants, or at a minimum the effects of the changes in taxes or prices on the weights are ignored. It is shown in Theorem 1 below that even if we relax the assumption of constant weights to permit a household's weight to depend on its income, having the weights not vary with prices results in very severe restrictions on household preferences and on social welfare functions.

The issue of whether utility weights are independent of prices is closely related to the issue of price independent welfare prescriptions for income distribution evaluation in Roberts (1980) and Slivinski (1983). For the additive Bergson class of welfare measures, Roberts (1980) shows that price independence requires that preferences take the Generalised Linear Gorman form of Muellbauer (1975, 1976) with limited variation in taste heterogeneity².

² On face value our theorem 1 below should be in Roberts (1980) or Slivinski (1983) but they do not look directly at $(\partial U / \partial v) (\partial v / \partial y)$. Rather, they want to find an equivalent income function $H(y)$ that is independent of prices. This is more like an 'aggregation over income' problem and hence leads to a PIGL solution (Gorman (1953) or Muellbauer (1975)).

Where prices vary across individuals Slivinski (1983) finds a further restriction is required – giving homothetic preferences. Here, we are interested in price independent utility weights for tax reform evaluation and the following theorem is equally restrictive. In particular, it implies that utility weights must depend on prices except in very special cases. This is relevant both because many analyses calculate or use utility weights θ_h or μ_h in ways that implicitly assume they are independent of prices, and because the second order approximation equations (7) and (9) depend on each utility weight's price elasticity. By Theorem 1 these elasticities are nonzero except in very special cases.

THEOREM 1. *Assume each household h has preferences that can be described by an indirect utility function $u_h = v_h(x_h, p)$, and that there exists a social welfare function in the form of equation (1). Then each household h has utility weight $\theta_h = s_h(x_h)$ for some function s_h that is independent of prices if and only if the social welfare function is*

$$\mathbf{U} = U[v_1(x_1, p), \dots, v_H(x_H, p)] = \sum_h [\kappa_h \ln x_h - a_h(p)] \quad (10)$$

for some functions a_h and constants κ_h .

COROLLARY. *Each household h has utility weights $\theta_h = s_h(x_h)$ for some functions s_h that are independent of prices if and only if each household h has a homothetic utility function and the utility weights are $\theta_h = s_h(x_h) = \kappa_h/x_h$.*

Proof of Theorem 1. See Appendix.

Note that by the definition of μ_h the same theorem and corollary apply replacing θ_h with μ_h , except that the μ_h utility weights are $\mu_h = \kappa_h$.

Homothetic indirect utility functions can always be written in the form $u_h = v_h(x_h, p) = x_h/\alpha_h(p)$ for some function $\alpha_h(p)$ that is homogenous of degree one in prices. Writing homothetic indirect utility functions in this way, Theorem 1 implies that the usual assumption of utility weights depending only on x_h requires the log linear social welfare function $\mathbf{U} = \sum_h \kappa_h \ln u_h$ with $\ln \alpha_h(p) = a_h(p)/\kappa_h$. The combination of log linear social welfare and homothetic preferences is very restrictive, and Theorem 1 shows that it can be avoided only by having households' utility weights depend on either prices or on the expenditures of other households.

Utility functions can only be identified up to an arbitrary monotonic transformation. To simplify welfare calculations, we therefore, always choose the representation of preferences that sets u_h equal to x_h in the base period for prices, for example, the standard representation of the Almost Ideal model has u_h equal to x_h when all prices are set equal to one. Therefore, for a given price regime the standard procedure of setting $U(u_1, \dots, u_H)$ equal to $U(x_1, \dots, x_H)$ and hence setting utility weights defined by $\theta_h = [\partial U(u_1, \dots, u_H)/\partial u_h] (\partial u_h/\partial x_h)$ equal to utility weights defined by $\theta_h = \partial U(x_1, \dots, x_H)/\partial x_h$ is legitimate, as long as it is recognised that these utility weights also depend on prices (except under the special circumstances defined by Theorem 1), and hence will change when prices change.

To illustrate these points, consider the Bergson (1938) class of social welfare functions (see Atkinson (1970)). Typically we might write social welfare as

$$\mathbf{U} = \sum_h \frac{v_h(x_h, p)^{1+\rho}}{1+\rho}, \quad (11)$$

in which ρ reflects the degree of inequality aversion. The weights become

$$\theta_h = v_h(x_h, p)^\rho \frac{\partial v_h(x_h, p)}{\partial x_h}, \quad (12)$$

which can be seen to depend on prices and requires complete information on individual utility $v_h(x_h, p)$ except when (11) reduces to (10).

In many applications welfare functions are expressed using income or total expenditures x_h in place of $v_h(x_h, p)$ to avoid measuring or estimating $v_h(x_h, p)$. To illustrate the effect of this substitution, consider the case in which preferences are from the PIGLOG class. This class covers the Almost Ideal system of Deaton and Muellbauer (1980) and the Exactly Aggregable Translog model of Jorgensen *et al.* (1982). PIGLOG indirect utility functions have the form

$$\ln v_h(x_h, p) = \frac{\ln x_h - \ln a_h(\mathbf{p})}{b_h(\mathbf{p})}, \quad (13)$$

where $a_h(\mathbf{p})$ and $b_h(\mathbf{p})$ represent price indices that reflect individual h 's substitution possibilities. At some base-period prices (where $\mathbf{p}^0 = \mathbf{1}$) $b_h(\mathbf{p}^0) = 1$ and $a_h(\mathbf{p}^0) = \alpha_h$. The α_h is some baseline equivalence scale for household h . As a result, at base prices, we may write

$$\theta_h^0 = \left(\frac{x_h}{\alpha_h} \right)^\rho, \quad (14)$$

which is a function of the equivalised total expenditures for each household. More generally, equation (14) can be obtained whenever the social welfare function is in the Atkinson class and utility functions have the Independence of Base (IB) property defined in Lewbel (1989).

This example shows that the use of suitably scaled $(x_h)^\rho$ utility weights is consistent with many popular demand models. Of course, it is still the case that the weights themselves will depend on prices and in general the $\partial \ln \theta_h / \partial \ln p$ term in (7) will not disappear.

III. MONEY METRIC MEASURES

There are many drawbacks associated with money-metric measures of utility and welfare. For example, Blackorby and Donaldson (1988) show that money metric measures generally violate concavity. Also the money metric social welfare function implies by its definition that the social marginal utility of income weights equal one for every household. Therefore, by Theorem 1, the money metric measure of social welfare either requires homothetic preferences for all households or it implicitly requires that social welfare depends on some functions of incomes and prices at other than the individual utility level.

Despite these substantial problems, money metric measures are commonly used as a substitute for (or approximation to) more formal welfare analyses. With these caveats, in this section we provide first and second order money metric welfare approximations which have an interestingly simple form.

Let $x_h = c_h(u_h, p)$ be the cost or expenditure function, which defines the total expenditure level required by household h to obtain the utility level u_h . Denote the Hicksian, or compensated demand function by $\partial c_h(u_h, p)/\partial p = q_h^c = q_h^c(u_h, p)$, which also equals $q_h(x_h, p)$.

The money metric measure of social welfare is the amount of money required to get every household back to the same utility level they had before the price or tax change. Total expenditures in the population are given by

$$X = \sum_h x_h = \sum_h c_h(u_h, p) \quad (15)$$

so the effect of a change in prices on total expenditures in the population X required to make every household as well off as before the price change is

$$\frac{\Delta X}{\Delta p} = \frac{\sum_h [c_h(u_h, p^*) - c_h(u_h, p)]}{\Delta p}. \quad (16)$$

Notice that for a price increase $\Delta X/\Delta p > 0$ while $\Delta U/\Delta p < 0$, since the former measures the increase in expenditures required to keep each household's utility level constant, while the latter measures the decrease in social welfare resulting from the price change. The money metric of social utility corresponds to the peculiar social welfare function that sets $\Delta U/\Delta p = -\Delta X/\Delta p$.

These effect of an infinitesimal change in prices on the X required to keep utilities unchanged is

$$\frac{\partial X}{\partial p} = \sum_h \frac{\partial c_h(u_h, p)}{\partial p} = \sum_h q_h^c, \quad (17)$$

which therefore yields the first order approximation of a price or tax change of

$$\Delta X/\Delta p \approx \partial X/\partial p = \sum_h q_h^c = \sum_h q_h.$$

This is the same as the first order approximation to the change in social welfare given earlier, in the case where all households have utility weights equal to one.

Applying the Taylor expansion as before gives the second order approximation

$$\frac{\Delta X}{\Delta p} \approx \sum_h q_h^c \left(1 + \frac{\Delta p}{2P} \frac{\partial \ln q_h^c}{\partial \ln p} \right). \quad (18)$$

The only difference between the second order welfare approximation (7) and the money metric approximation (18) is that the latter replaces each utility weight θ_h with one, and the compensated (Hicksian) own price elasticity appears instead of the ordinary uncompensated (Marshallian) own price elasticity.

Since compensated own price elasticities are always negative, we can

unequivocally sign the bias in the first order approximation of the money metric measure of price effects, in that the second order approximation is always smaller in magnitude than the standard first order approximation.

As before, slightly different approximations can be obtained using log prices and budget shares. These first and second order approximations are

$$\frac{\Delta X}{\Delta p} = \frac{\Delta \ln p}{\Delta p} \frac{\Delta X}{\Delta \ln p} \approx \frac{\Delta \ln p}{\Delta p} \frac{\partial X}{\partial \ln p} = -\frac{\Delta \ln p}{\Delta p} \sum_h x_h w_h \quad (19)$$

and

$$\frac{\Delta X}{\Delta p} \approx -\frac{\Delta \ln p}{\Delta p} \sum_h x_h w_h \left(1 + \frac{\Delta \ln p}{2} \frac{\partial \ln w_h^c}{\partial \ln p} \right), \quad (20)$$

which is identical to equation (9) where the alternative utility weights μ_h are set equal to x_h and the compensated budget share elasticity replaces the corresponding uncompensated elasticity.

IV. AN EMPIRICAL APPLICATION

To describe consumer behaviour we will use the Quadratic Almost Ideal model derived in Banks *et al.* (1996). This is a rank three budget share system that is quadratic in the logarithm of total expenditure – having the attractive property of allowing goods to have the characteristics of luxuries at low levels of total expenditure, say, and necessities at higher levels. These quadratic terms are found to be empirically important in describing household budget behaviour in the United Kingdom. The indirect utility function for this model is of the form

$$\ln v = \left\{ \left[\frac{\ln m - \ln a_h(\mathbf{p})}{b_h(\mathbf{p})} \right]^{-1} + l_h(\mathbf{p}) \right\}^{-1}, \quad (21)$$

where $a_h(\mathbf{p})$ has the Translog form and, $b_h(\mathbf{p})$ and $l_h(\mathbf{p})$, are differentiable, homogenous of degree zero functions of prices. When $l_h(\mathbf{p})$ is set to zero indirect utilities are simply PIGLOG as in (13) above.

Choosing

$$\ln a_h(\mathbf{p}) = \alpha_0 + \sum_{i=1}^n \alpha_{hi} \ln p_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} \ln p_i \ln p_j, \quad (22)$$

$$b_h(\mathbf{p}) = \prod_{i=1}^n p_i^{\beta_{hi}} \quad (23)$$

and

$$l_h(\mathbf{p}) = \sum_{i=1}^n \lambda_{hi} \ln p_i \quad (24)$$

and using Shephard's Lemma yields the budget share equations for household h (with total expenditure x_h), given by

$$w_{hi} = \alpha_{hi} + \sum_{j=1}^n \gamma_{ij} \ln p_j + \beta_{hi} \ln \left[\frac{x_h}{a_h(\mathbf{p})} \right] + \frac{\lambda_{hi}}{b_h(\mathbf{p})} \left\{ \ln \left[\frac{x_h}{a_h(\mathbf{p})} \right] \right\}^2 \quad (25)$$

where $\sum_j \gamma_{ij} = 0$, $\sum_i \alpha_{hi} = 1$, $\sum_i \beta_{hj} = 0$ and $\sum_i \lambda_{hi} = 0$ to ensure homogeneity, symmetry and adding-up.

This model is estimated in Banks *et al.* (1994) using UK *Family Expenditure*

Survey data for the period 1970–86. The budget system is defined over five goods – food, fuel, clothing, alcohol and ‘other non-durable goods’ – and the sample is restricted to non-retired married couples without children where the head is employed and the household lives in London or South East England. These households are selected to form a reasonably homogeneous group in order to reduce the number of additional demographic factors which need to be controlled for in estimating preferences. Model parameters are estimated using the whole sample (4,785 observations over 68 quarterly price points) and elasticities are computed for each household. However, the welfare analysis that follows is carried out using only those observations observed in the final year of our data.

The tax change we choose to illustrate these approximations is a 17.5% tax on clothing. This represents a large price change, but is within the bounds of possibility in government tax reform. Indeed 17.5% is the current rate of Value Added (Sales) Tax in the United Kingdom but many groups of goods (including childrens clothing) are exempt; hence proposed moves towards a uniform expenditure tax would require tax changes of this magnitude. For each of the four cases below we plot first and second order approximation errors against log expenditure for each household for the 17.5% change in price. In addition, at the end of this section, we will also show how each approximation improves as tax or price changes become smaller.

The first approximations we consider are the first and second order approximations to $\Delta U/\Delta p$ presented in Figs. 1 and 2. To do this we need to

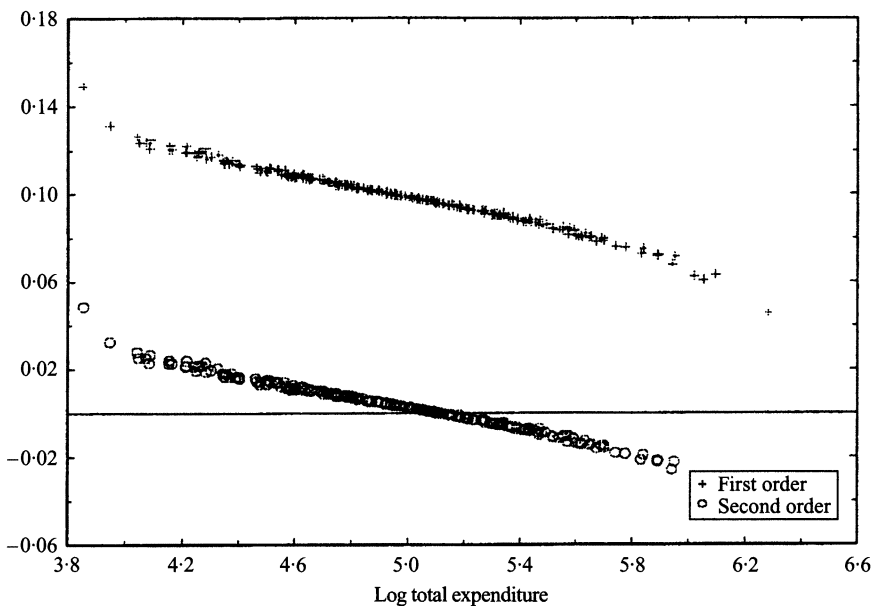


Fig. 1. $\Delta U/\Delta p$ – Approximation errors as proportion of true change.

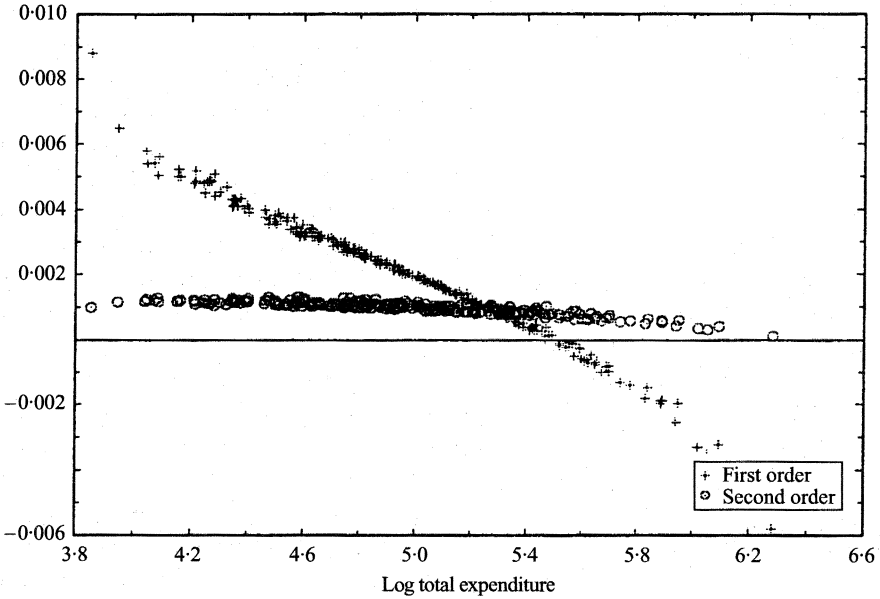


Fig. 2. $\Delta U/\Delta p$ - Approximation errors as proportion of true change (log case).

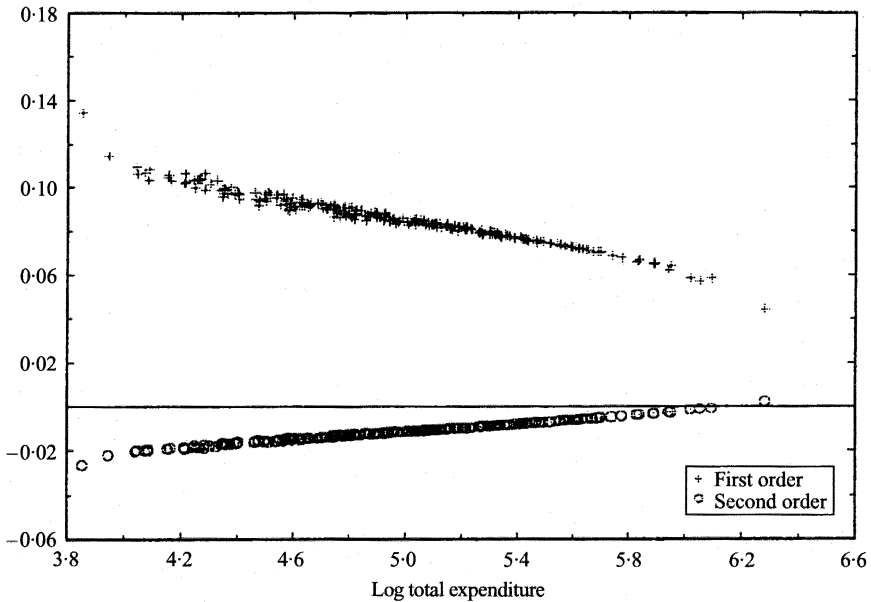


Fig. 3. $\Delta X/\Delta p$ - Approximation errors as proportion of true change.

parameterise (1) to compute utility weights for the approximations in (4) and (7). To do this we use the social welfare function defined in (11) to weight the household indirect utilities given by (21). In both these figures the inequality aversion parameter, ρ , is set to zero so the social welfare function simply sums household indirect utilities. Each figure shows the first and second order

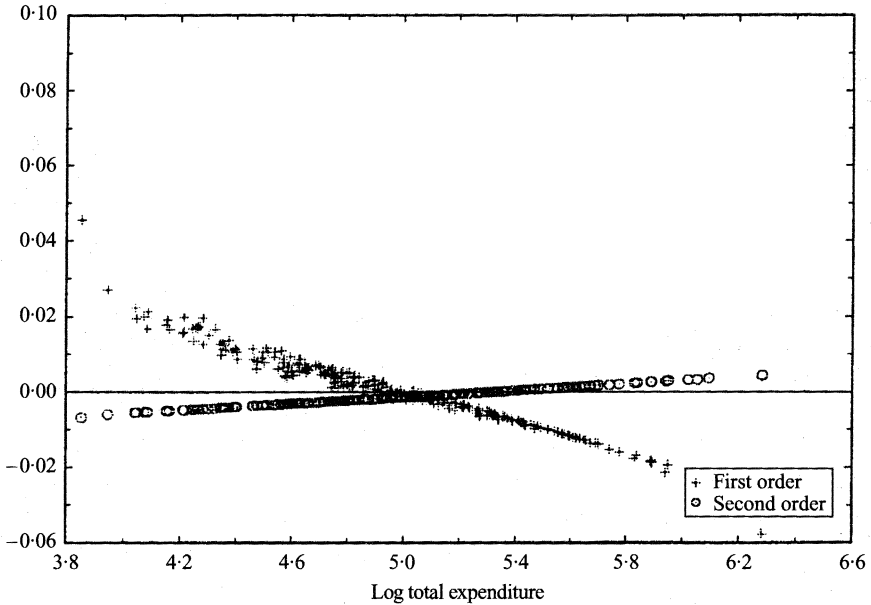


Fig. 4. $\Delta X/\Delta p$ – Approximation errors as proportion of true change (log case).

approximation errors expressed as a proportion of the true change in the households' indirect utility, $v_h(x_{h0}, p_1) - v_h(x_{h0}, p_0)$, plotted against log total household expenditure.

Fig. 1 shows that the magnitude of the first order approximation error in $\Delta U/\Delta p$ is considerably greater than that of the second order error. On average the first order error is 9.9% whereas the addition of the second order term in the approximation reduces this average error to 0.3% of the true welfare change. First order approximations using the log approximation work better, although the units of the approximation error have changed. The superiority of the first order approximation in logs may be due in part to the lack of the systematic bias that was demonstrated for the first order linear approximation. Fig. 2 shows that for some households the first order approximation error is now negative and the average first order error is substantially reduced (to 1.3% of the true welfare change). The second order approximation is now upwardly biased for every household but the size of this bias is very small.

In Figs 3 and 4 we look at approximations to the money metric measure of welfare loss outlined in Section III above – again using an example reform of 17.5% tax on clothing. Each figure now shows the first and second order approximation errors expressed as a proportion of the true change in the cost function, $c(u_{h0}, p_1) - c(u_{h0}, p_0)$ against log expenditure for each household in the sample. Once again the magnitude of first order approximation error in Fig. 3 – the approximation to $\Delta X/\Delta p$ – is much bigger than that of the second order error. The first order case gives an average approximation error of 8.5% whilst the second order average error is -1.2%. Notice that, in addition, the first order error is positive for every household as predicted in Section III. It is not

possible to sign the bias in the second order error due to the elasticity switching between greater and less than one. Finally, Fig. 4 shows that, in direct correspondence to the linear case, for the log approximation measure in (20) the first order approximation behaves much better on average whilst still being some way off for households at either end of the expenditure distribution. In this case the average first order error turns out to be only 0.03% of the true change.

Table 1 gives population summary statistics for each approximation for different size price changes. In each case the summary statistics for the 17.5% price change correspond to the aggregate measures of the household effects illustrated in Figs 1–4. In addition, however, we present summary statistics for $\Delta U/\Delta p$ where we have set the inequality aversion parameter in the social welfare function to be quite high ($\rho = -3$). As expected all approximations improve as price or tax changes become smaller. Average second order approximation errors are less than first order errors in all cases except for log approximation for $\Delta X/\Delta p$ where individual household errors are much bigger (as reflected in the standard deviation) but just happen to average to zero for this dataset (see Fig. 4).

V. CONCLUSIONS

There is an obvious attraction to simply using information on observed commodity demands to assess the welfare implications of tax reform. No response parameters are required and therefore the analysis can transcend misspecification of preferences and is not subject to estimation error in own or cross-price demand elasticities. However, tax reforms are often far from marginal and can involve a significant realignment of relative prices. In such cases we have shown that second-order approximations that involve derivatives of demand equations can produce improvements in welfare measurement – highlighting the usefulness of reliable estimates of price and income elasticities.

In certain popular cases we have shown that the difference between first and second order approximations is the same sign for every individual, and therefore, will not average out in any standard aggregate social welfare measure. For a tax reform that adds a new group of goods to the tax base in the United Kingdom at a tax rate of 10% we find that the bias can be of the order of 5%. For smaller reforms we show that suitable first order approximations can work very well. However, the second-order approximations in our examples uniformly produce improvements in the measurement of changes in aggregate social welfare.

Our results on the comparison of approximations are relative to a set of estimated demand elasticities. The estimation of the elasticities themselves is a different problem. For example, one could non-parametrically estimate the elasticities to use in the approximations above. In the absence of micro-data one possibility is to use an aggregate estimator of the demand elasticity in the second order correction term. However, there is no guarantee that the aggregate statistic will estimate the average of the individual elasticities without bias.

Table I
Approximation Errors as Proportion of True Change in Welfare

	$\Delta\phi = 17.5\%$		$\Delta\phi = 10.0\%$		$\Delta\phi = 5\%$		$\Delta\phi = 2.5\%$	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\Delta U/\Delta\phi(\rho = 0)$								
First order	0.099	0.013	0.057	0.008	0.029	0.004	0.015	0.002
Second order	0.003	0.012	0.004	0.007	0.003	0.004	0.002	0.002
$\Delta U/\Delta\phi(\rho = -3)$								
First order	0.080	0.014	0.046	0.008	0.023	0.004	0.012	0.002
Second order	-0.016	0.012	-0.007	0.007	-0.003	0.004	-0.001	0.002
$\Delta U/\Delta\phi(\log, \rho = 0)$								
First order	0.013	0.012	0.008	0.007	0.004	0.004	0.002	0.002
Second order	0.006	0.001	0.004	0.001	0.002	0.000	0.001	0.000
$\Delta X/\Delta\phi$								
First order	0.085	0.011	0.049	0.006	0.025	0.003	0.012	0.002
Second order	-0.012	0.004	-0.004	0.002	-0.001	0.001	-0.000	0.000
$\Delta X/\Delta\phi(\log)$								
First order	-0.000	0.011	-0.000	0.007	-0.000	0.003	-0.000	0.002
Second order	-0.004	0.002	-0.002	0.001	-0.001	0.001	0.000	0.000

Examination of the second order approximation formulas derived here provides information on the adequacy of the different approximations. For example, the smaller are the price elasticities of quantities and utility weights, the better are the standard first order approximations. The closer the price elasticities of the quantities and utility weights are to constants, the better are second order approximations. The first order approximation based on the change in prices has a systematic bias that is not present in the first order approximation based on the change in log prices. Even if the true functional form for demands is unknown, if it is believed to be, say, close to quadratic in log prices, that would suggest using a second order approximation in log prices for welfare analysis.

In this paper we have analysed the quality of approximations defined in terms of ordinary utility functions. This may not be the most relevant metric – for example the success of an environmental tax might be defined in terms of its success in reducing demand for pollutants. We have shown that the first order approximations hold quantities fixed so, at a minimum, second order approximations would be needed to calculate the effects of environmental taxes intelligently. More generally, price elasticity estimates are required for any sensible measurement when the object of interest is changes in quantity demanded rather than change in utility.

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APPENDIX. *Proof of Theorem 1*

Integrate the expression $\theta_h = s_h(x_h) = \partial U[v_1(x_1, p), \dots, v_H(x_H, p)] / \partial x_h$ with respect to x_h for each h from 1 to H to get the expression for the social welfare function $U[v_1(x_1, p), \dots, v_H(x_H, p)] = \sum_h S_h(x_h) + A(p)$ for some function of prices $A(p)$, when the function $S_h(x_h)$, which is independent of all prices, is defined by $\partial S_h(x_h) / \partial x_h = s_h(x_h)$. Since each x_h appears only in the function v_h , the above expression for the social welfare function implies that $U[v_1(x_1, p), \dots, v_H(x_H, p)] = \sum_h \Psi_h[v_h(x_h, p)]$ and $\Psi_h[v_h(x_h, p)] = S_h(x_h) + a_h(p)$, where $a_h(p)$ are some functions of prices satisfying $\sum_h a_h(p) = A(p)$. Indirect utility functions are homogeneous of degree zero in prices and total expenditures, so $\Psi_h[v_h(x_h, p)]$ must be homogenous of degree zero in prices and total expenditures, and therefore $\partial \Psi_h[v_h(x_h, p)] / \partial x_h = \partial S_h(x_h) / \partial x_h = s_h(x_h)$ is homogeneous of degree minus one in prices and total expenditures, but $s_h(x_h)$ is independent of prices by assumption, and the only function of x_h alone that is homogeneous of degree minus one is $s_h(x_h) = \kappa_h / x_h$ for some constant κ_h . It follows that $S_h(x_h) = \kappa_h \ln x_h$ (plus an arbitrary constant of integration that, without loss of generality, can depend on h and can be absorbed into $a_h(p)$) so $\Psi_h[v_h(x_h, p)] = \kappa_h \ln x_h + a_h(p)$, which proves the theorem. Corollary 1 follows from the definition of homothetic indirect utility and the definition of the utility weights.