Review of:
Nonlinear Dynamics: a Two-Way Trip from Physics to Math

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Perhaps surprisingly, given its origins in celestial mechanics, modern nonlinear dynamics has never made it into the core physics curriculum. As the authors of *Nonlinear Dynamics: a Two-Way Trip from Physics to Math* observe in the Preface, quantum mechanics, which also has its origins around the turn of the century, caught the attention of the physics community and quantum mechanics deals with linear operators on linear (Hilbert) spaces after all. In some of the more phenomenological areas of physics, however, nonlinear dynamics has found fertile soil: fluid dynamics, laser dynamics, plasma physics, mesoscopic physics (quantum chaos), to name a few. The authors of the present book are active in the field of laser dynamics, and it shows.

Reflecting the increasing interest in the application of dynamical systems to virtually every field of research, recent years have seen a proliferation of books pitched at the advanced undergraduate/first-year graduate level. Books presenting the key elements of the theory are, for example, Rasband [1], Hale & Koçak [2], Ott [3], Strogatz [4], Glendinning [5] and Verhulst [6]. Besides these general texts there is a good helping of books aimed at specialised audiences of a more pure (e.g., Palis & Takens [7]) or more applied (e.g., Moon [8], Schuster [9]) flavour.

The book under review is directed to beginning graduate students and researchers in physics, mathematics and engineering, but will mostly appeal to physicists. It is meant to be a comprehensive text. In 16 chapters the 350 pages or so cover a wide range of topics. The somewhat enigmatic subtitle, as explained in the Preface, refers to the hand-in-hand approach of mathematics and physics in the evolution of problems and solutions (i.e., theories) in our sciencific inquiries. The pairs classical mechanics-calculus (Newton), quantum mechanics-Hilbert spaces (Von Neumann) and nonlinear dynamics-topology (Poincaré) serve as examples.

The style of the book is informal and people should not be put off by the message on the back cover that "The book presents rigorous results..." A fair selection of key results is presented in the form of theorems but proofs are omitted (or sketched, in rare cases). References to the literature are given instead. The organisation of the material is well thought-out and each chapter ends with a brief summary.

The story begins in Chapter 1 with a description of some experiments illustrating that physical systems can show markedly different behaviour as parameters (physical control variables) are varied. As examples we are offered turbulent fluids, modulated lasers and symmetry-breaking plasmas. In the next six chapters the elementary theory of dynamical systems is presented: linear dynamics, limit sets, invariant manifolds, Poincaré sections, (structural, Lyapunov) stability, two-dimensional flows and bifurcations. By then we have also had a closer look at some mathematical models underlying the phenomena reviewed in Chapter 1. The chapter on two-dimensional flows essentially deals with the Poincaré-Bendixson Theorem classifying the possible limit sets of such systems. Some unnecessary confusion is caused by the fact that there are two results presented under the name Poincaré-Bendixson Theorem, one a lemma to the other.

Chapter 7 on bifurcations first introduces centre manifold reduction and normal forms and then gives a survey of the simple local codimension-one bifurcations. All of this for maps as well as flows. Relatively much time is spent on discussing the delicate Hopf bifurcation for maps (treatment of the strongly resonant cases is deferred till after the introduction of averaging in Chapter 14). This bifurcation, which also goes under the name of Naimark-Sacker bifurcation, is called a codimension-two bifurcation by the authors, while only its codimension-one cases are treated. Indeed, codimension-two bifurcations receive hardly any attention at all in the book.

Chapter 8 presents results of numerical simulations of some low-dimensional continuous and discrete systems focussing on period-doubling cascades, mode-locking, the Lorenz attractor and crisis. Here we have our first encounter with chaotic dynamics, loosely defined in the text as the aperiodic behaviour observed in numerical experiments. Chaos is not popularised in the book. Rather, the authors are interested in the intricate structure of periodic orbits underlying complex behaviour and, in the following four chapters, quickly move on to discussing homoclinic tangles, horseshoes, one-dimensional maps, knots and braids. This part of the book is topological/combinatorial in nature. The chapter on the application of the techniques of knots and braids, the authors' own expertise, is one of the best parts of the book. The subject is still the domain of a rather specialised group of researchers (many of them working in laser dynamics) and is not found in many other books. The methods of knots and braids can be very powerful in predicting the existence of certain periodic orbits, much like kneading theory does in unimodal maps. Unfortunately, they are in large part restricted to three-dimensional flows with globally defined Poincaré sections, which in practise often means periodically perturbed planar systems.

The final four chapters cover such miscellaneous subjects as experimental time series, Lyapunov exponents, dimensions and entropy, averaging (used to study subharmonic resonance and strongly resonant Hopf bifurcation), bifurcations and symmetries, and global bifurcations. The last chapter on global bifurcations takes up the line of the Chapters 9-12 and treats horseshoe creation in systems with Šil'nikov-type bifurcations, both with and without symmetry.

My main criticism is that, partly due to the wide spread of material, the discussion tends to remain too superficial for a textbook. This is made worse by the fact that the text contains an insufficient number of examples, which otherwise could have given the book a more explicit character. For instance, there are no examples to illustrate the Poincaré-Bendixson and Hopf bifurcation results (either for maps or flows). Instead, the pages are richly sprinkled with exercises, to which parts of the main line of development are relegated. The authors seem to find encouragement for this strategy in a quote by Von Humboldt, but the mathematically inclined or the not so easily convinced might be less quickly reassured. Those will benefit from reading one or two other books simultaneously. On the other hand, the laser models thrown in at several places provide useful non-standard illustrations of some interesting dynamical phenomena (subharmonic resonance, Šil'nikov bifurcation).

Despite the book's wide scope, some areas of nonlinear dynamics, inevitably, go uncovered. There is virtually nothing on conservative (Hamiltonian) systems and (multi)fractals (although Cantor sets are discussed). Nothing will be found on numerical methods (apart from time series analysis), control of chaos, PDEs or infinite dimensional dynamical systems, pattern formation, complexity or quantum chaos. Since dynamical systems theory is the geometrical approach to differential equations (and iterated maps), figures can be extremely helpful in conveying ideas. In this respect the book is sufficiently illustrated. There is an adequate list of references.

In conclusion, as someone versed in nonlinear dynamics (and with a background in physics) I enjoyed reading the book, partly because an intuitive account often goes down better than a detailed mathematical exposition. The student, though, might quickly be seen grabbing at mathematically more explicit accompanying material.

References

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[3] Ott, E., Chaos in Dynamical Systems (Cambridge University Press, Cambridge, 1993).

[4] Strogatz, S., Nonlinear Dynamics and Chaos: with applications to physics, biology, chemistry, and engineering (Addison-Wesley, Reading, 1994).

[5] Glendinning, P., Stability, Instability and Chaos: an introduction to the theory of nonlinear differential equations (Cambridge University Press, Cambridge, 1994).

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[8] Moon, F.C., Chaotic and Fractal Dynamics (Wiley, New York, 1992).

[9] Schuster, H.G., Deterministic Chaos: an introduction (3rd ed.) (VCH, Weinheim, 1995).