# E466 APPLIED MECHANICS

# DYNAMICS COURSEWORK

## An Investigation of Vibrations of Bending Beams and Rotating Shafts by Exact and Finite-Element Methods

## 1. OBJECTIVES

a) To induce familiarity with the free vibration behaviour of simple beams and shafts subject to a variety of end conditions.

b) To induce familiarity with the finite-element method and to assess the accuracy of this method for calculating modal frequencies and modelling mode shapes.

c) To compare calculations based on consistent-mass and lumped-mass finite-element modelling.

## 2. TORSIONAL VIBRATIONS OF CIRCULAR SHAFTS

Consider a uniform steel shaft of length L=1 m and circular cross-section ($\rho=7.8\times10^3$ kg/m$^3$, $G=7.9\times10^{10}$ N/m$^2$, $I=4\times10^{-7}$ m$^4$). Using **Matlab**, compute for each of the following boundary conditions the **fundamental modal frequency** of 1-element, 2-element, 3-element and 4-element approximations in the **consistent-mass** description, and compare the results with the **exact** frequency:

**a) Free-Free Shaft**
**b) Fixed-Free Shaft**
**c) Fixed-Fixed Shaft**

In each case sketch the first three **mode shapes** as given by solutions of the wave equation. Comment on possible rigid body modes.

## 3. BENDING VIBRATIONS OF SIMPLE BEAMS

Now consider a uniform steel beam of length L=5 m ($\rho=7.8\times10^3$ kg/m$^3$, $E=2.1\times10^{11}$ N/m$^2$, $I=8\times10^{-3}$ m$^4$, $A=1\times10^{-2}$ m$^2$) in the following situations:

### a) Propped Cantilever

Investigate the **rate of convergence** of the first six **modal frequencies** of the finite-element model as the number of elements is increased. Consider both the

**consistent-mass** and the **lumped-mass** (without rotational inertia) approach. **Plot your results** to enable comparison. Show how the results compare with the **Euler-Bernoulli** solutions (use Matlab's `fzero' function to solve the characteristic equation).

Sketch the first six **mode shapes** as given by the Euler-Bernoulli theory.

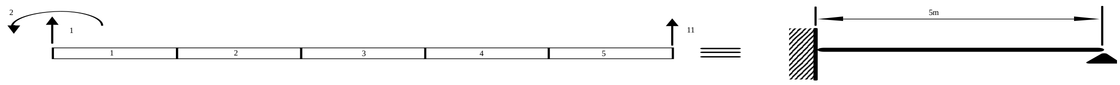


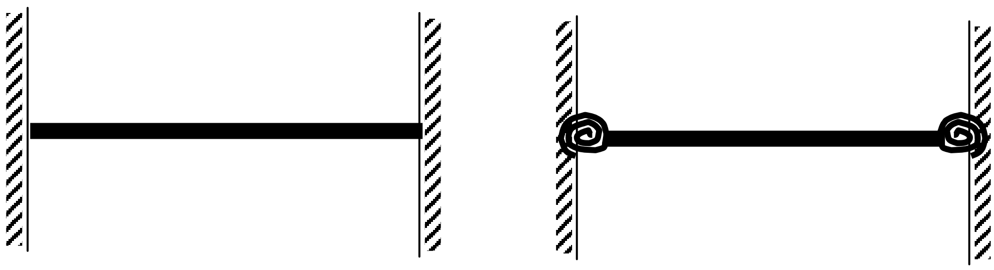
*Figure 1. 5-element propped cantilever; element length = 1 m; rigid degrees of freedom: 1, 2 and 11.*

**b) Fixed-Fixed Beam**

Repeat the above for this case.

**c) Beam with Flexible End Support**

Replace the fully restrained support in b) by a pair of rotational springs (see Fig. 2) of equal stiffness k ranging from 0 Nm/rad to very high stiffness.



*Fixed-fixed beam* *Beam with flexible end support*

*Figure 2*

Observe the effects of this on the value of the fundamental frequency and mode shape in an 8-element idealisation**. Plot variation of fundamental frequency against stiffness.** The finite-element mode shape may be viewed against the Euler-Bernoulli simply-supported mode shape to aid in assessing the variations in shape, particularly near the end supports. **Comment on this.** What can you say about the limit $k \rightarrow \infty$?

**d) Multi-Span Beam**

A simply-supported two-span beam with a 2:1 ratio for the spans is modelled by two finite elements, one for each span. Investigate the effect of the intermediate support on the **fundamental frequency** of the beam. Consider also a few other span ratios while keeping the total length of the beam constant at 3L (=15 m). Interpret the (discrete) mode shapes.

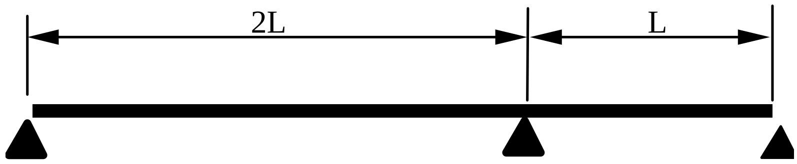



*Figure 3. Two-span beam*

## 4. REPORT REQUIREMENTS

- A report is to be submitted by **Wednesday 31 January 2007** describing the investigations conducted, the results obtained and the conclusions drawn.
- It must contain the derivations of the exact solutions used.
- Be careful to give accurate references (**with citation marks in your main text**) for the information you include.
- The report should be concise and self-contained, and its length should not exceed 30 pages.

GH/06