Network theory and analysis of football strategies

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Disclaimer

- Joint work with Hugo Touchette
- ((Very) Pure) Mathematician speaking
- For any Americans in the audience:

$$\text{Football} = \text{Soccer}$$

What can maths say about football?

Mathematicians are good at two things:

- Finding patterns
- Turning easy things intro abstract nonsense

(Normally we do it the other way around)

Question

Can the abstract nonsense tell us something useful?

The Fundamental Theorem of football

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Theorem (Fundamental Theorem of football)

Good football teams have a recognizable style

- But not necessarily *the same* for all teams!

Question

- *Can we describe the "style" mathematically?*
- *And then say something about the team?*

What to focus on?

Many aspects of football one might look at!

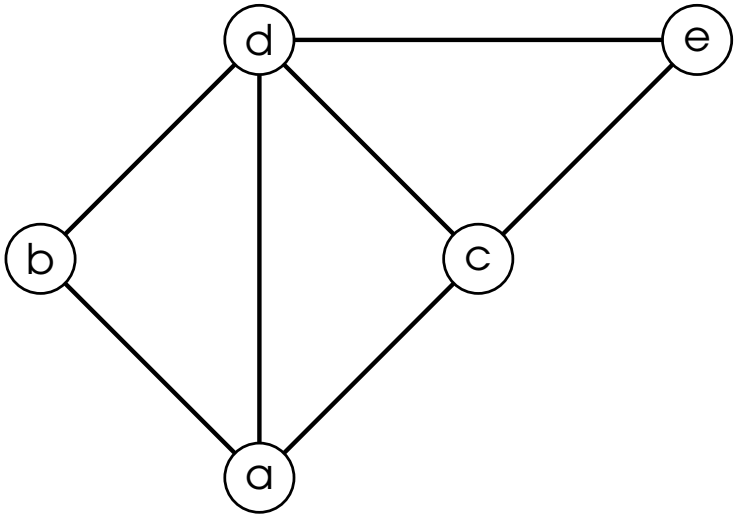
- Goals
- Fouls
- Percentage of victories
- Ball possession
- Passing information

We'll focus on the last one

A bit of abstract nonsense: Networks

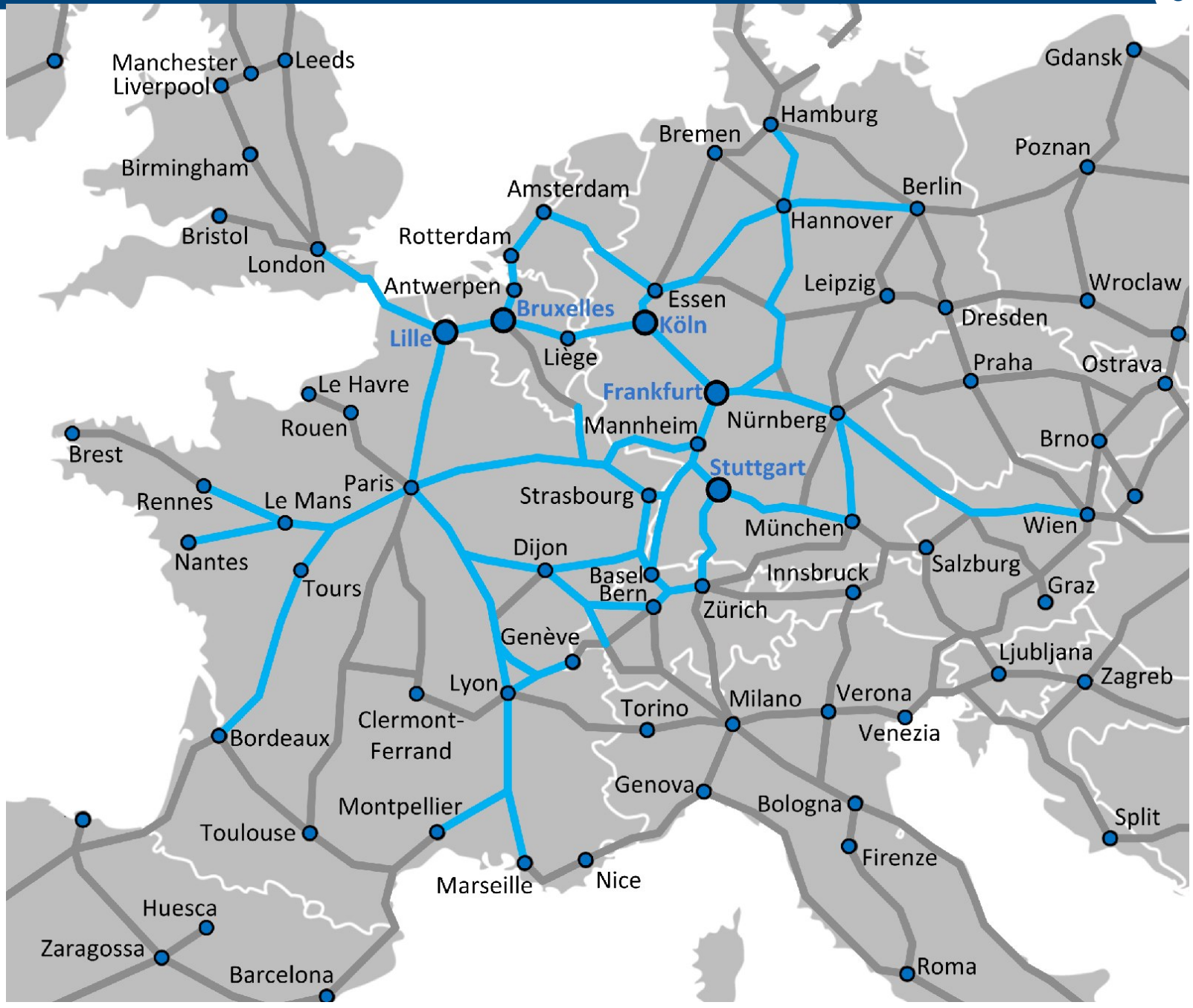
A network consists of:

- A collection of nodes (or vertices)
- Some edges connecting the nodes



- Nodes can have a clear physical meaning.
- But they don't have to.

Example: High speed train network



Example: North America power grid

Example: The Internet

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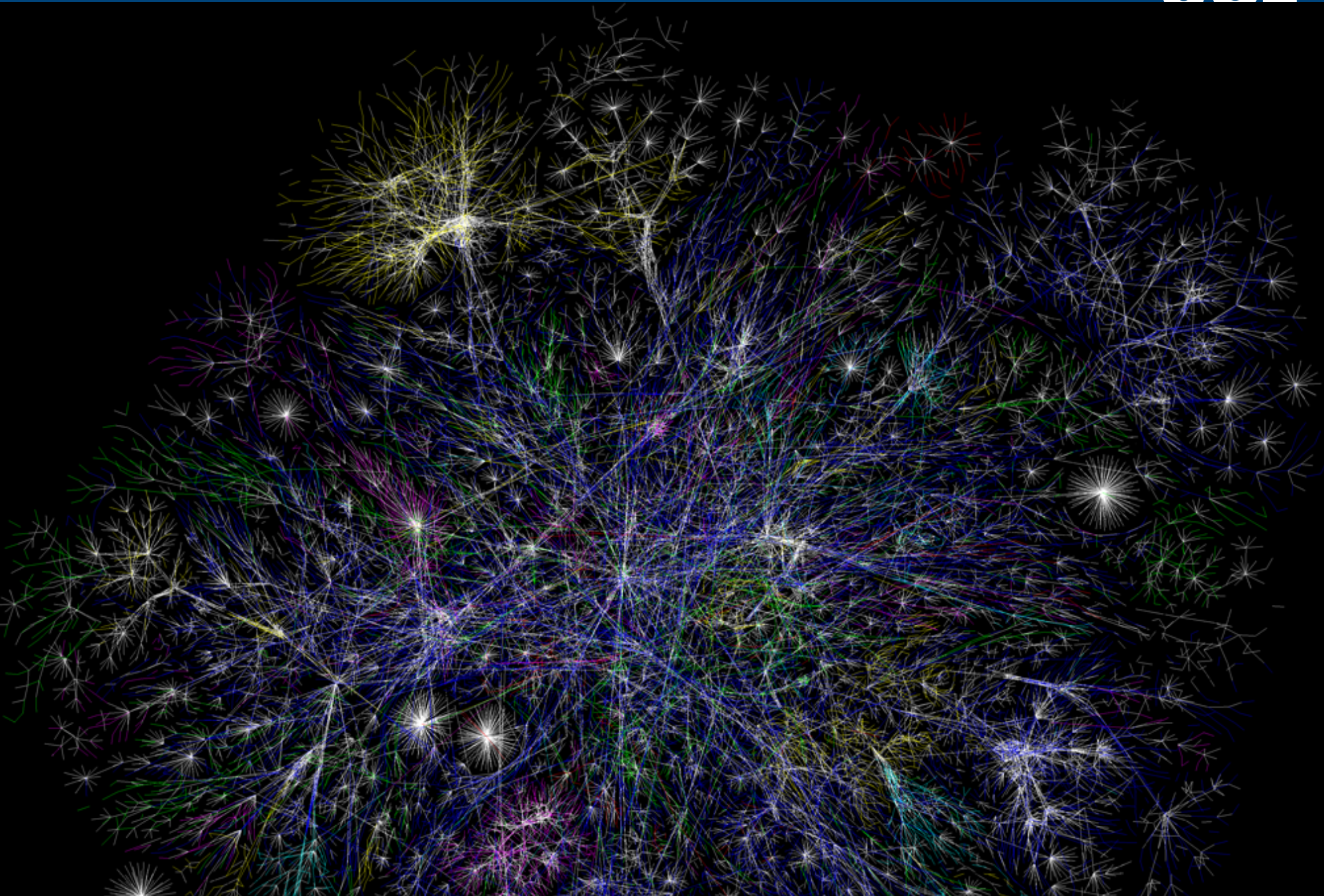
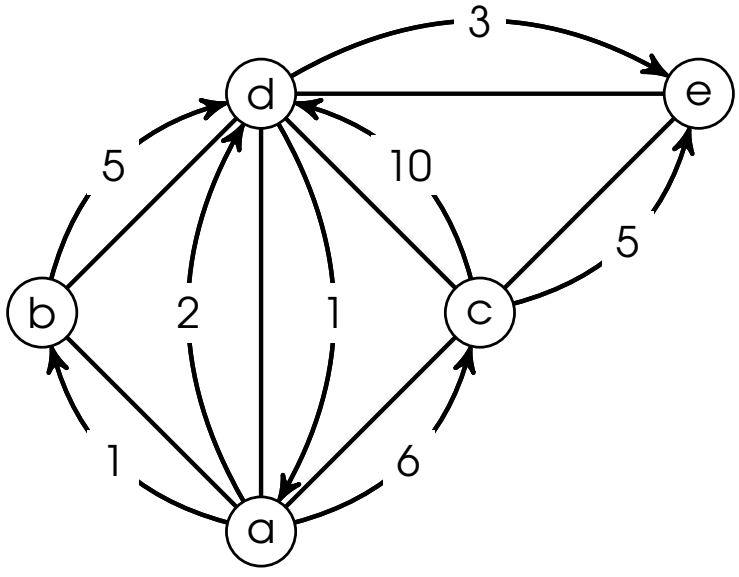


Oriented networks

Not all edges are created equal!

- We can use directed edges (or arrows)
- Perhaps pointing in both directions
- Or attach weights to them



The passing network of a football team

We associate a network to each football team

- Nodes are the team players
- Arrows represent passes between the players
- Weights given by the number of passes

In the drawing, represent the weight as arrow thickness

The passing network of a football team

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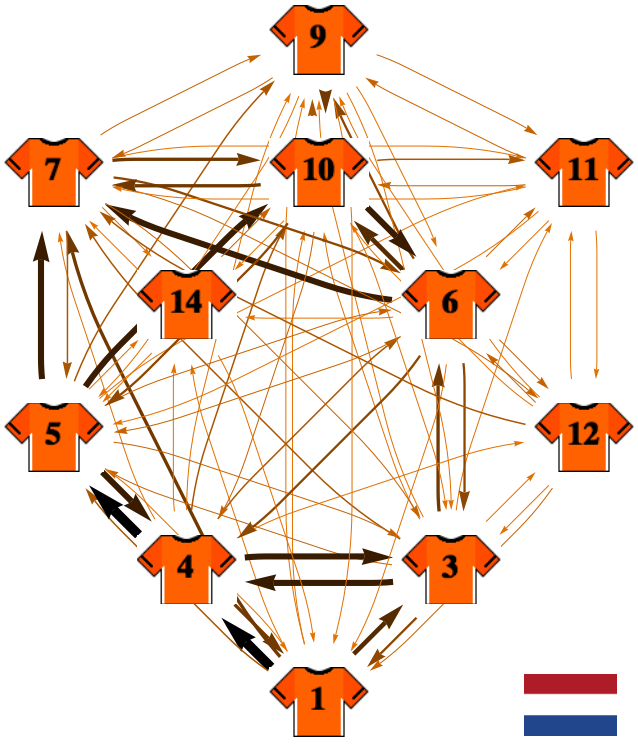
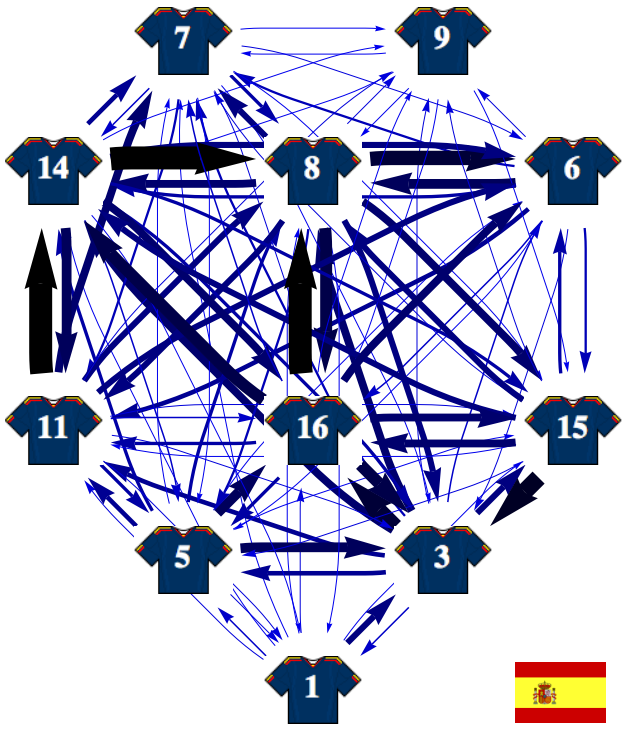
Netherlands vs. Spain





The passing network of a football team

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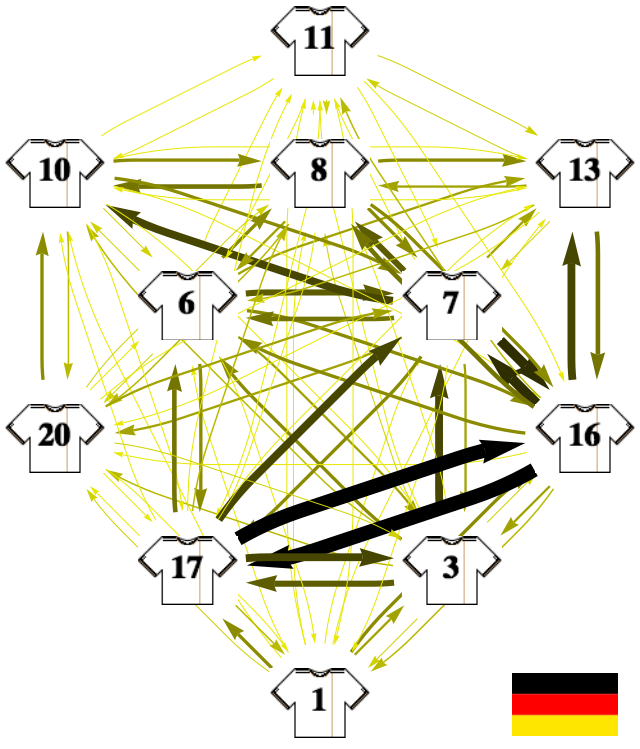
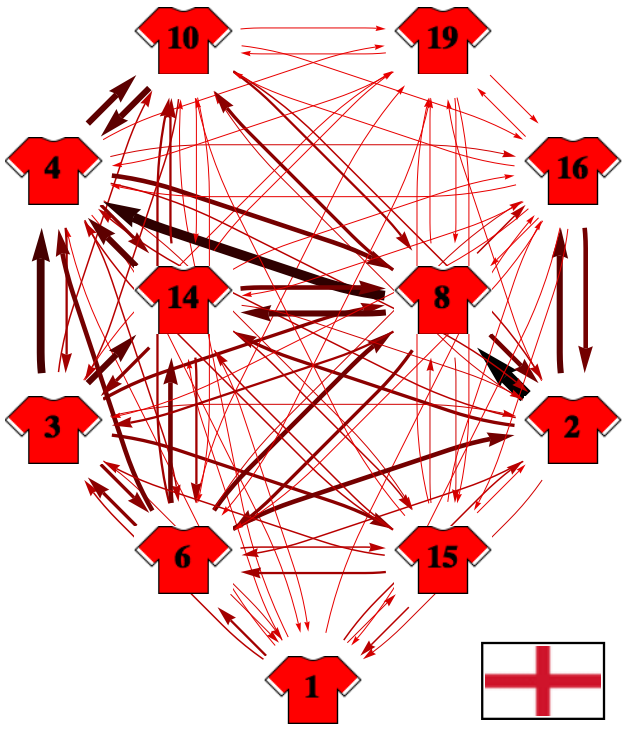
Germany vs. England





Extracting information from the network

Mathematical representation of the network

- Use the adjacency matrix (A_{ij})

$$A_{ij} = \text{Number of passes from } i \text{ to } j$$

- Matrix is bad for visualization
- But good for computations

How an adjacency matrix looks like

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$$
\text{Spain} \rightarrow \begin{pmatrix}
0 & 7 & 3 & 0 & 1 & 1 & 0 & 2 & 1 & 1 & 2 \\
3 & 0 & 6 & 3 & 4 & 8 & 2 & 6 & 8 & 7 & 11 \\
2 & 9 & 0 & 1 & 1 & 5 & 0 & 6 & 5 & 1 & 9 \\
0 & 2 & 1 & 0 & 4 & 9 & 2 & 6 & 5 & 4 & 3 \\
0 & 1 & 1 & 2 & 0 & 5 & 1 & 4 & 3 & 1 & 2 \\
1 & 7 & 4 & 11 & 7 & 0 & 2 & 7 & 8 & 8 & 10 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\
2 & 2 & 5 & 7 & 8 & 7 & 2 & 0 & 14 & 2 & 3 \\
1 & 5 & 3 & 7 & 6 & 14 & 1 & 9 & 0 & 5 & 8 \\
0 & 12 & 2 & 5 & 2 & 7 & 2 & 1 & 7 & 0 & 8 \\
0 & 10 & 5 & 8 & 3 & 14 & 1 & 5 & 10 & 8 & 0
\end{pmatrix}
$$

About the players: centrality

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Question

How to measure the importance of a node in a network?

Answer: Centrality measures

- There are different ways of measuring importance
- Different types of centrality to address them!

Closeness centrality

- Mean distance from a node to the other ones
- Distance is the inverse of the number of passes

$$C_i = \frac{20}{\sum_{j \neq i} \frac{1}{A_{ij}+1} + \sum_{j \neq i} \frac{1}{A_{ji}+1}} - 1$$

- w and $1 - w$ are weights to passing/receiving
- There is some normalization going on
- Actual value is not important
- Just focus on the relative order

Pagerank centrality

- Recursive notion of "popularity"
- A node is popular if linked by other popular nodes

$$x_i = p \sum_j A_{ji} \frac{x_j}{k_j^{\text{out}}} + (1 - p)$$

- $k_j^{\text{out}} = \sum_i A_{ji} =$ total number of passes made by j
- p is the (estimated) probability of passing the ball
- Estimate made by heuristics
- $p = 0.85$ normally works well

Betweenness centrality

- How the network suffers when a node is removed
- A node is popular if linked by other popular nodes

$$C_B(i) = \frac{1}{10^2} \sum_{j,k \neq i} \frac{d_{jk}(i)}{d_{jk}}$$

- d_{jk} = distance from j to k
- $d_{jk}(i)$ = distance without going through i
- Nodes with high C_B are dangerous for the network

Centralities for Spanish players

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	Player	Closeness	Pagerank	Betweenness
1	Casillas	0.672	5.47%	0
3	Piqué	3.347	8.96%	1.19
5	Puyol	1.849	8.89%	0.92
6	Iniesta	1.889	8.35%	0.12
7	Villa	1.798	10.17%	1.19
8	Xavi	4.358	10.26%	2.49
9	Torres	0.578	8.30%	0
11	Capdevilla	2.975	8.96%	1.19
14	Alonso	3.742	10.26%	2.49
15	Ramos	2.251	10.17%	1.19
16	Busquets	3.239	10.17%	1.19

What do network tell us?

- Different teams have very different networks
- Quick overview of a team style
  - Most used areas of the court
  - Short distance or long distance passes
  - Players not participating enough
  - Problems between players
- Centrality measures give information about players
- Plenty of useful information for a coach!

The limits of the tool

Network analysis is not a silver bullet

- Not for all sports
- Only tracks *successful* passes
  - Add a probability to the weight!
- Doesn't account for shots and goals
  - Add an extra node for the opponent's gate!
- What happens when a player gets changed?
- Passing data is hard to obtain!

Thanks for your attention!