Invariance under twisting

Javier López Peña

Algebra Department
University of Granada (Spain)

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- Pascual Jara,
- Florin Panaite,
- Fred Van Oystaeyen

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Outline

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Drinfeld twist

- H bialgebra, $F \in H \otimes H$ a 2–cocycle.
- H_F new bialgebra:
  - Same algebra structure as H,
  - Comultiplication $\Delta_F(h) := F\Delta(h)F^{-1}$.
- A an H–module algebra.
- $A_{F^{-1}}$ new algebra with $a * a' := (G^1 \cdot a)(G^2 \cdot a')$ (being $F^{-1} := G^1 \otimes G^2$).

Theorem (Majid, 1997)

$A_{F^{-1}}$ is an H_F–module algebra, and

$$A_{F^{-1}} \# H_F \cong A \# H$$

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Drinfeld Double

- $(H, r = r^1 \otimes r^2)$ a f. dim. quasitriangular Hopf algebra.
- $\mathcal{D}(H)$ the Drinfeld double of H:
  - $\mathcal{D}(H) = H^{*coop} \otimes H$ as a coalgebra.
  - Product $(p \otimes h)(p' \otimes h') := p(h_1 \rightharpoonup p' \leftharpoonup S^{-1}(h_3)) \otimes h_2 h'$ (where $\rightharpoonup$ and $\leftharpoonup$ are the regular actions)
- $\underline{H}^*$ a left H–module algebra structure in H^* given by

$$h \cdot \varphi := h_1 \rightharpoonup \varphi \leftharpoonup S^{-1}(h_2)$$

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"Unbraiding" of braid product

- (H, r) a quasitriangular Hopf algebra
- $H^+, H^- \leq H$ Hopf subalgebras with $r \in H^+ \otimes H^-$
- B a right H^+–mod alg. C a right H^-–mod alg.
- $B \underline{\otimes} C$ their braided product wrt $c \otimes b \mapsto br^1 \otimes cr^2$
- $\pi : H^+ \# B \to B$ alg map with $\pi(1\#b) = b$

Theorem (Fiore-Steinacker-Wess, 2003)

The map $\theta : C \to B \otimes C$ given by $\theta(c) := \pi(r^1\#1) \otimes cr^2$ is an alg. map from C to $B\underline{\otimes}C$ and $B\underline{\otimes}C \cong B \otimes C$.

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What have these results in common?

$$A_{F^{-1}}\#H_F \cong A\#H \qquad \mathcal{D}(H) \cong \underline{H}^*\#H$$
$$B\underline{\otimes}C \cong B \otimes C \qquad A\#H \cong A \otimes H$$

- Two algebras X and Y
- A "product" Z of X and Y
- A "deformation" $\overline{X}$ of X
- A "product" $\widetilde{Z}$ of $\overline{X}$ and Y
- An algebra isomorphism $X \cong \widetilde{X}$

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Is it possible to find a general result giving us all the former isomorphisms?

The Answer: Yes, but first, we should clarify what do we mean by “product” and “deformation”...

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Outline

What do we mean by "product"?

Definition (Cap-Schichl-Vanžura'94, Van Daele'94, . . .)

Z is a ***twisted tensor product*** of X and Y if there exist a linear map $R : Y \otimes X \longrightarrow X \otimes Y$ such that Z is isomorphic to $X \otimes Y$ endowed with the product

$$\mu_R := (\mu_X \otimes \mu_Y) \circ (X \otimes R \otimes Y)$$

Equiv. to conditions given in prof. Schneider's talk:

- $i_X : X \hookrightarrow Z$ and $i_Y : Y \hookrightarrow Z$ injective algebra maps.
- The map $x \otimes y \longmapsto i_X(x) \cdot i_Y(y)$ is a linear isomorphism.

The origin of the story: "Distributive laws", by J. Beck

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The maps for our examples

All the algebras in our examples are twisted tensor products:

Drinfeld twist $A\#H = A \otimes_R H$ with $R(h \otimes a) := h_1 \cdot a \otimes h_2$.

Drinfeld double $\mathcal{D}(H) = H^* \otimes_R H$ with
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Informal Definition

By a ***deformation*** of an algebra A we mean:

- Some datum (maps, other algebras,...) associated to A
- A new product defined in A upon this datum.

That is, we build a new product, keeping the old vector space.

Remark

This is an ***inner deformation***, by contrast to ***outer deformations*** like Gerstenhaber's formal deformation.

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This is an ***inner deformation***, by contrast to ***outer deformations*** like Gerstenhaber's formal deformation.

What do we mean by "deformation"?

Informal Definition

By a ***deformation*** of an algebra A we mean:

- Some datum (maps, other algebras, . . .) associated to A
- A new product defined in A upon this datum.

That is, we build a new product, keeping the old vector space.

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This is an ***inner deformation***, by contrast to ***outer deformations*** like Gerstenhaber's formal deformation.

Construction of our deformation I

1. A, B algebras
2. $R : B \otimes A \to A \otimes B$ linear map
3. Linear maps $\mu : B \otimes A \to A$ and $\rho : A \to A \otimes B$
4. Define $* : A \otimes A \to A$ by $* := m_A \circ (A \otimes \mu) \circ (\rho \otimes A)$
5. Assume the (technical and boring) compatibility conditions:
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Remark

Former datum is a generalization of W. Ferrer and B. Torrecillas ***left-right twisting datum***.

Our first two examples fit into this deformation scheme:

Drinfeld twist: $\mu(h \otimes a) := h \cdot a$, $\rho(a) := G^1 \cdot a \otimes G^2$. Associated product is $a * a' = (G^1 \cdot a)(G^2 \cdot a')$, giving $A_{F^{-1}}$.

Drinfeld double: $\mu(h \otimes \varphi) := h_1 \rightharpoonup \varphi \leftharpoonup S^{-1}(h_2)$, $\rho(\varphi) := \varphi \leftharpoonup S^{-1}(r^1) \otimes r^2$, associated product $\varphi * \varphi' = (\varphi \leftharpoonup S^{-1}(r^1))(r_1^2 \rightharpoonup \varphi' \leftharpoonup S^{-1}(r_2^2))$, as in $\underline{H}^*$.

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Outline

Invariance under twisting: Theorem I

- A, B algebras,
- (R, μ, ρ) left-right twisting datum with R twisting map.
- $\lambda : A \to A \otimes B$ linear map such that
  - $\lambda(1) = 1 \otimes 1$,
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- A^d the deformation of A.

Theorem

$R^d := (A^d \otimes m_B) \circ (\lambda \otimes m_B) \circ (R \otimes B) \circ (B \otimes \rho)$ is a twisting map, and $(A \otimes m_B) \circ (\rho \otimes B)$ is an algebra isomorphism between $A \otimes_R B$ and $A^d \otimes_{R^d} B$.

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Consequences

The strong points of our theorem:

- It recovers the isomorphisms in our first two examples.
- The isomorphism is explicitly given.

And the weak ones...

- Last two examples don't fit.
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- $A \otimes_R B$ a twisted tensor product
- A' another algebra structure on A
- $\rho : A' \to A \otimes_R B$ an algebra map
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Invariance Theorem II

- This theorem generalizes the former one
- It also contains the last two examples:

Unbraiding: $\lambda(c) := \pi(u^1\#1) \otimes c \cdot u^2,$

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Can this theorem be of any use?

Possible ways of taking advantage of the Invariance Theorem:

- Use it to relate two different twisted tensor products.
  Could help with the classification, up to isomorphism, of factorization structures
- Explicitly build a deformation in the terms of the theorem in order to build a new object isomorphic to the original one.
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Final remarks

1. Most of the results can be translated to (strict) monoidal categories
2. Under suitable conditions, the Invariance Theorem can be iterated (cf (JLPVO)).

Moral

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