MATHEMATICS 0054 (Analytical Dynamics)
YEAR 2023–2024, TERM 2

PROBLEM SET #1

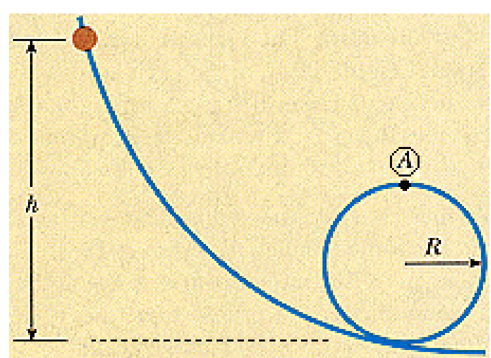
This problem set is due at the *beginning* of class on Thursday 18 January.

Topics: Review of the fundamental principles of Newtonian mechanics; Galileo's principle of relativity. Review of solvable cases of one-dimensional motion. Systems of particles and conservation laws: linear momentum, angular momentum, energy (internal and external potentials).

Readings:

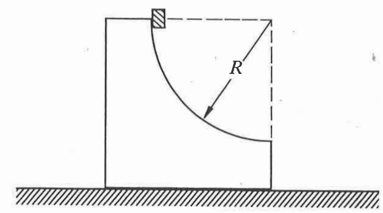
- Handout #1: Newton's First Law and the Principle of Relativity.
- Handout #2: Newton's Second Law.
- Handout #3: Newton's Third Law.
- Handout #4: How to Do Mechanics Problems.
- Morin, Chapter 1 (Strategies for solving problems [handout].
- Handout #5: Solvable Cases of One-Dimensional Motion.
- Handout #6: Momentum, Angular Momentum, and Energy; Conservation Laws.
- Feynman, Section 4–1 (What is energy?) [handout].
- Kleppner and Kolenkow, Section 1.9 (Motion in plane polar coordinates) and Chapter 9 (Central force motion) [handouts].
- Handout #7: More on Central-Force Motion.

1. A bead of mass m (with a hole through its center) is threaded on a wire ("loop-the-loop") as shown in the diagram to the right. It starts from rest at height h and slides frictionlessly down the wire. Find the magnitude and direction of the normal force N exerted by the wire on the bead when the bead arrives at the point A located at the top of the circle.



   (*Hint:* First use conservation of energy to find the bead's speed when it reaches point A. Then use $\mathbf{F} = m\mathbf{a}$ to find the normal force exerted by the wire on the bead. You will need to recall the kinematics of nonuniform circular motion: when a particle moves in a circle of radius R with speed $v(t)$, the radial component of its acceleration is v^2/R inwards, and the tangential component of its acceleration is dv/dt.)

2. A small cube of mass m slides frictionlessly down a quarter-circular path cut into a large block of mass M, starting at rest from the top of the path. The large block slides frictionlessly on a horizontal table, also starting from rest. Find the velocity v of the small cube relative to the earth at the moment it leaves the block.



(*Hint:* Use conservation of energy and conservation of momentum. *Clue to check correctness of your answer:* When $m = M$, $v = \sqrt{gR}$.)

3. A mass m whirls on a frictionless table, held to circular motion by a string that passes through a hole in the table. The string is pulled *very slowly* so that the radius of the circle gradually decreases from ℓ_1 to ℓ_2. Show that the work done in pulling the string equals the increase in kinetic energy of the mass.

   (*Hint:* Use conservation of angular momentum to find the mass' speed v when the string has length ℓ, assuming that it had speed v_1 when the string had length ℓ_1. Using this, find the tension in the string when the string has length ℓ. Integrate this from $\ell = \ell_1$ to $\ell = \ell_2$ to find the work done.)

4. A skier of mass m starts from rest at the top of a large hemispherical hill of radius R, and slides down frictionlessly. Measure angles θ from the vertical, and measure potential energy from the top. Find:

   (a) the skier's potential energy as a function of angle;

   (b) the skier's kinetic energy as a function of angle;

   (c) the skier's speed as a function of angle;

   (d) the radial and tangential components of the skier's acceleration, as a function of angle;

   (e) the angle at which the skier flies off the hill.

   [*Hint for part (d):* You will need once again the kinematics of nonuniform circular motion, as discussed in the hint to Problem 1. The radial acceleration is easy to compute here, using your answer from part (c). There are at least two different approaches to finding the tangential acceleration: Probably the simplest is to apply $\mathbf{F} = m\mathbf{a}$, using tilted (radial-tangential) axes. Alternatively, use $a_{\tan} = dv/dt$ together with the chain rule

$$\frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = \frac{dv}{d\theta}\frac{v}{R} = \frac{1}{R}\frac{d}{d\theta}\left(\frac{1}{2}v^2\right) .$$

   (But make sure you understand every step in this chain of equalities.) You might find it enjoyable to compute the tangential acceleration in *both* of these (very different) ways, and see if you get the same answer. *Hint for part (e):* First find the normal force as a function of θ, using $\mathbf{F} = m\mathbf{a}$ combined with your answers from part (d).]